The Welfare Effects of Bank Liquidity and Capital Requirements*

Skander J. Van den Heuvel† and Sergio Villalvazo‡

May 2026

Abstract

The stringency of bank liquidity and capital requirements should depend on their social costs and benefits. Equilibrium asset returns yield two simple formulas that express the welfare cost of each requirement as a function of observable variables only. Based on U.S. data, the welfare cost of a 10% liquidity requirement is equivalent to a permanent loss in consumption of about 0.02%, a modest impact. Even using conservative measurements, the cost of a similarly sized increase in the capital requirement is roughly tenfold. Still, optimal policy relies on both requirements to mitigate moral hazard from deposit insurance and is close to Basel III levels.

*We thank Toni Ahnert, William Bassett, Francesca Carapella, Francisco Covas, Burcu Duygan-Bump, Antonio Falato, Sigurd Galaasen, Pedro Gete, Itay Goldstein, Gary Gorton, Gazi Kara, Agnese Leonello, David Martinez-Miera, Mark Mink, Thien Nguyen, Ettore Panetti, Ned Prescott, David Rappoport, Harald Uhlig, Alex Vardoulakis, and seminar participants at CEBRA, Cleveland Fed, Columbia, ECB, Fed Board, FDIC, FIRS, IMF, NBER, SAET, SED, and the Wharton Conference on Liquidity and Financial Crises for valuable comments, and Sorelle Peat, Jacob Fahringer, Olamide Bola, Tristan D'Orsaneo, Oliver Hyman-Metzger, Colin Stokes, and Dulce Lopez Cruz for expert research assistance. Skander Van den Heuvel gratefully acknowledges that a substantial part of this research was conducted during a secondment at the ECB. The views expressed here do not necessarily represent the views of the Federal Reserve Board, the ECB, or their staffs.

†Federal Reserve Board, skander.j.vandenheuvel@frb.gov

‡Federal Reserve Board

1 Introduction

The global financial crisis spurred key financial reforms, including the strengthening of bank capital requirements and the introduction of new liquidity requirements, as part of Basel III. Even so, an important debate continues on the question of whether the strengthening of these requirements has been appropriate, excessive, or insufficient, and their calibration remains at the top of the regulatory agenda.[1] While there is widespread agreement that capital requirements can and have helped make banks safer and that liquidity stress exacerbated the crisis through runs and fire sales, the ongoing debate in large part reflects differing views about the existence and magnitude of costs to society from imposing restrictions on banks' balance sheets. While some progress has been made for capital regulation, the social cost of liquidity regulation and its interaction with capital regulation are much less well understood. Some have argued for narrow banking, where deposits are backed exclusively by safe, liquid assets - akin to a 100% liquidity requirement.[2] The harm from liquidity stress would presumably be greatly reduced, if not eliminated, if such a policy were adopted. But what would be the cost? Clearly, to determine the optimal levels of liquidity and capital requirements the question of their social cost must be addressed.

This paper argues that liquidity and capital regulations can each impose an important cost for a similar reason: they reduce the ability of banks to create net liquidity through the transformation of illiquid loans into liquid deposits – a key, traditional function of banks. After all, capital requirements directly limit the fraction of bank loans that can be financed by issuing liquid, deposit-like liabilities. Liquidity requirements force banks to hold safe, liquid assets against deposits, limiting their liquidity transformation by restricting the asset side of their balance sheet. This can impose a social cost because safe, liquid assets are necessarily in limited supply and have competing uses (see, for example, Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood, Hanson, and Stein (2015)).

More specifically, the contribution of this paper is threefold. First, it builds a framework to analyze the social costs and benefits of liquidity and capital requirements. It provides a rationale for their joint use and characterizes the best division of labor in order to foster financial stability in the least costly way. Second, it derives two simple formulas for the

[1] Banking agencies have recently proposed changes to capital regulation that, if adopted, would decrease common equity tier 1 requirements by 5 percent for the largest banks and by 8 percent for smaller banks; see https://www.federalreserve.gov/newsevents/pressreleases/bcreg20260319a.htm.

[2] A classic reference is Friedman (1960). See Cochrane (2014) for a more recent proposal. Gorton et al. (2022) note the similarity between the liquidity coverage ratio (LCR), a key Basel III liquidity requirement, and narrow banking. They argue that the historical experience from the U.S. National Banking Era suggests that narrow banking is unlikely to be desirable. We examine narrow banking in section 5.4.

magnitude of the equilibrium welfare costs of capital and liquidity requirements. These formulas are functions of observable variables only. The third contribution is quantitative: the paper deploys these formulas, using U.S. data, in order to measure the welfare cost of each requirement. Based on a full-model calibration, we also quantify their welfare benefits. To the best of our knowledge, this is the first study to quantify the welfare costs and benefits of liquidity requirements.

The framework, based on Van den Heuvel (2008), embeds liquidity-creating banks in an otherwise standard general equilibrium growth model. Due to their role in the provision of liquidity services, bank liabilities are special and, as a result, the Modigliani-Miller theorem fails to hold for banks: their capital structure is not irrelevant. The welfare costs of the capital and liquidity requirements depend crucially on the value of the liquidity provided by bank deposits and by government bonds – a safe and liquid alternative to bank deposits that can also be used by banks to satisfy liquidity regulation. A key insight is that equilibrium financial spreads reveal the strength of preferences for liquidity and this allows us to quantify the welfare costs of each requirement without imposing restrictive assumptions on these preferences. Furthermore, the analysis shows how regulation can affect capital accumulation and the size of the banking sector, as credit provision can shift to nonbanks due to regulatory arbitrage, depending on how special bank loans are. The formulas for the welfare costs take these general equilibrium feedbacks into account. The model also incorporates a rationale for the use of both regulations, based on a moral hazard problem created by deposit insurance (or similar types of government guarantees), which if unchecked, can lead banks to take on excessive credit and liquidity risk.

The main findings are as follows. The preference for liquidity implies that the pecuniary returns on liquid assets – bank deposits and government bonds – are lower than the returns on non-liquid assets – equity in the model. For banks, this departure from Modigliani-Miller can result in a binding capital requirement. The liquidity requirement – a minimum ratio of banks' holdings of government bonds to their deposit liabilities – binds if the convenience yield on government bonds exceeds the convenience yield on bank deposits, net of the noninterest cost of servicing those deposits. Because of competition, banks pass on the cheap deposit funding to borrowers in the form of a lower lending rate. However, if binding, both the capital requirement and the liquidity requirement limit the extent of this pass-through. Possible noninterest costs can also increase the lending rate. If the net impact of these factors is such that bank loans are still relatively inexpensive, firms will borrow exclusively from banks. Otherwise, the equilibrium will be one of both bank and nonbank credit, and the share of banks in total credit will be determined endogenously.

As a consequence, in the model, liquidity and capital regulation can each lead to migration of financial activity to nonbank financial intermediaries, such as shadow banks, or to disintermediation. The extent of this migration depends in part on how special bank loans are. For liquidity regulation, a shift to nonbanks is more likely if the supply of high-quality liquid assets is low relative to the demand for such assets, so that their convenience yield is high. Moreover, these regulations can alter not only the composition of the financial sector, but also overall economic activity, through their effect on firm investment.

Turning to normative results, both capital and liquidity regulations are helpful in mitigating the moral hazard from deposit insurance and thereby preventing financial crises. First, moral hazard can lead banks to take on excessive credit risk. A capital requirement is helpful in limiting this problem by ensuring that shareholders internalize potential losses. In contrast, a liquidity requirement does not mitigate credit risk and may exacerbate it. Second, moral hazard can lead banks to take on excessive liquidity risk, a problem that is mitigated by the liquidity requirement. Thus, the model suggests a simple division of labor: liquidity regulation should address liquidity risk, and capital regulation should address credit risk.

These benefits are not a free lunch, however, as the regulations also entail social costs. If binding, each requirement reduces banks' ability to perform liquidity transformation, a socially valuable activity. The model can be used as a lens to see how the magnitude of these costs can be measured with real-world data. With equilibrium asset returns revealing the strength of investors' preferences for liquidity, two simple formulas that are functions of observable variables only are derived for the welfare costs of the regulations. First, the marginal welfare cost (MWC) of the liquidity requirement scales with the *difference* between the convenience yield on government bonds (Treasuries) and the convenience yield on bank deposits, and therefore with the spread between these two instruments. Specifically,

$$\text{MWC of Liquidity Req.} \approx \frac{\text{Deposits}}{\text{Consumption}}(\text{Deposit-Treasury Spread})$$

(There is an adjustment for the net noninterest costs of servicing deposits, which raises the spread, as shown in proposition 2.) The intuition for this result is as follows. The liquidity requirement essentially removes Treasuries from nonbank investors and puts them in banks – but banks can finance these new assets with deposits which, like Treasuries, also provide liquidity services. This entails a net social cost only to the extent that the liquidity services of bank deposits (net of their noninterest costs) are, at the margin, valued less than the liquidity services of Treasuries to nonbank investors, a difference that is revealed by their spread.

Second, the marginal welfare cost of the capital requirement scales with the convenience yield on bank deposits (adjusted for noninterest costs and the liquidity requirement, as shown in proposition 3):

$$\text{MWC of Capital Req.} = \frac{\text{Bank Loans}}{\text{Consumption}} \times \text{Adjusted Convenience Yield on Deposits}$$

This expression reflects the finding that the capital requirement constrains the ability of banks to issue deposits in equilibrium.

We then use U.S. data to measure the cost-revealing financial spreads and the other variables in the formulas. The welfare cost of a 10 percent liquidity requirement is found to be equivalent to a permanent loss in consumption of about 0.02%, a modest cost.[3] Even using a conservative method, the cost of a similarly sized increase in the capital requirement is found to be roughly ten times as large. The cost of a complete move to narrow banking would be another order of magnitude higher, about 2.4% of consumption.

Why is the cost of the liquidity requirement so much smaller than the cost of the capital requirement? Fundamentally, the liquidity requirement has a much smaller impact on *net* liquidity creation. As mentioned, from the perspective of nonbank investors, a liquidity requirement effectively transforms government bonds into deposits, both instruments prized for their liquidity. Accordingly, its welfare cost reflects the *difference* between those two convenience yields, which shows up as the deposit-Treasury spread in the above formula. Unsurprisingly, this spread is small in the data. In contrast, the capital requirement reduces bank deposits and (simplifying significantly) replaces them with illiquid equity. Hence, its welfare cost scales with the convenience yield on deposits, not its difference relative to some other convenience yield, resulting in a greater equilibrium cost. While this explanation simplifies considerably by abstracting from many general equilibrium feedbacks and other factors, it conveys the root cause of the cost differential.

Based on a full calibration and global numerical solution of the model, we also quantify the welfare benefits of the regulations and compare them to the costs. This exercise indicates that the post-crisis reforms to capital and liquidity requirements have resulted in a net increase in welfare of about 0.2% of consumption. Echoing the costs, the welfare benefits of the liquidity requirement are far smaller than for the capital requirement, and socially

[3] This is for a liquidity requirement that is modelled after the liquidity coverage ratio (LCR), one of two liquidity rules introduced by Basel III. The other, the net stable funding ratio (NSFR), is outside the scope of this paper.

optimal requirements are found to be close to Basel III levels.

The numerical solution also allows us to evaluate the quality of first-order approximations of the costs of discrete changes in requirements that are based on the formulas for their *marginal* welfare costs. We find that the exact and approximate numbers are close, within 1 to 3 basis points of each other for changes of up to 10 percentage points in each requirement. Adjusting for this difference does not alter the main conclusions.

As a caveat, the model does not feature a lender of last resort that could save solvent banks with liquidity problems, which could lessen the need for ex-ante liquidity regulation. Because of that, the analysis may overstate the beneficial role of liquidity regulation, but this does not matter for the results on the cost. That said, in reality, the lender of last resort function of central banks is not completely free of challenges. Deciding whether a bank only experiences liquidity problems or liquidity and solvency problems can be difficult in crisis times. And it has been argued that interventions by a lender of last resort can themselves lead to moral hazard problems (see, e.g., Farhi and Tirole (2012)). To the extent that the lender of last resort function entails economic costs, these could be compared to the costs of liquidity regulation, which this paper attempts to quantify.[4]

Related literature Several recent papers present quantitative, macroeconomic models of optimal bank capital regulation, including Begenau (2020), Begenau and Landvoigt (2022), Clerc et al. (2015), Elenev, Landvoigt, and Van Nieuwerburgh (2021), Martinez-Miera and Suarez (2014), and Nguyen (2015).[5] In their calibrated versions, these models each yield an interior level of the capital requirement that maximizes a welfare criterion, with the optimal levels ranging from 6 percent in Elenev et al., whose model features financially constrained producers, to 16 percent in Begenau and Landvoigt, whose model features unregulated as well as regulated banks. There are three main differences with the model developed here.

First, the above-mentioned papers do not aim to examine liquidity requirements, which is a focus in this paper. Second, the reason Modigliani-Miller fails for banks is different, except for Begenau (2020) and Begenau and Landvoigt (2022), in which, as in this paper, it fails chiefly because of the liquidity of bank debt.[6] Third, these studies all rely on a full-

[4]Carlson et al. (2015) discuss the relation between liquidity regulation and the lender of last resort.

[5]In addition, recent studies also provide quantitative examinations of optimal *time-varying* capital requirements; see, e.g., Canzoneri et al. (2020), Davydiuk (2017), and Malherbe (2020). See also Jermann and Xiang (2024), who highlight a time-inconsistency problem for setting time-varying capital requirements when deposits are sticky and (partially) uninsured.

[6]This is also the key friction in Gorton and Winton (2017) and Van den Heuvel (2008), who also show that bank capital requirements can have a social cost because they reduce the ability of banks to create liquidity in equilibrium. In addition, in Elenev at al. (2021)'s model, the specialness of bank debt as a safe asset is one among four frictions that lead to a failure of Modigliani-Miller at banks.

model calibration to draw out quantitative implications, which is not necessary to compute the welfare costs of regulation in this paper. Instead, we derive explicit formulas that can be used to quantify these costs only with a handful of observable data.

In other words, our approach identifies the right sufficient statistics that reveal the welfare costs of regulation in equilibrium. According to Chetty (2009), this method "combines the advantages of reduced-form empirics –transparent and credible identification– with an important advantage of structural models – the ability to make precise statements about welfare." This could be viewed as attractive, and complementary to full-model calibration, in the context of macroeconomic models with financial intermediation, because such models tend to have many parameters that are difficult to calibrate or estimate. Dávila (2019) and Dávila and Goldstein (2023) are recent papers that also do not rely on a full-model calibration to derive effects of financial regulations. Sraer and Thesmar (2018) use similar methods to make inferences about aggregate counterfactuals based on firm-level, reduced-form estimates. All that said, we do complement our approach with a full-model calibration to evaluate the model's nonlinearities and quantify the regulatory benefits.

Finally, there is an emerging literature on the theoretical benefits of liquidity requirements, based on preventing bank runs or fire sales, including, for example, Calomiris, Heider, and Hoerova (2015), Diamond and Kashyap (2016), Kara and Ozsoy (2019), Kashyap, Tsomocos, and Vardoulakis (2024), and Vives (2014). Quantitative, positive examinations of the effects of liquidity and capital requirements are presented by Corbae and D'Erasmo (2021), who examine their effects on bank risk-taking, market structure, efficiency, and stability in a model of industry dynamics, by Covas and Driscoll (2014), who introduce these requirements into a DSGE model and by De Nicolò, Gamba, and Lucchetta (2014), who take a micro-prudential perspective.

Organization of the paper The next section presents a baseline version of the model with minimal assumptions and analyzes the agents' decision problems. A positive analysis of general equilibrium follows in section 3, and section 4 presents the formulas for the social costs of regulation. These are used in section 5 to measure the welfare costs of liquidity and capital requirements, as well as a hypothetical government-imposed switch to narrow banking. Section 6 presents an expanded version of the model with bank risks and moral hazard from deposit insurance. It examines the benefits of the liquidity and capital requirements in mitigating the harm from bank failures and other adverse externalities from excessive risk taking. Crucially, this section will show that the formulas for the welfare *costs* remain valid with moral hazard and bank failures. The final section concludes.

2 The Baseline Model

As mentioned, the model extends Van den Heuvel (2008), which adds two features to the standard growth model: first, households have a need for liquidity and, second, certain institutions, labelled banks, are able to create financial assets, deposits, that provide liquidity services. As a novel element in this model relative to its precursor, government bonds can also serve as liquid assets for households and businesses, similar to Krishnamurthy and Vissing-Jorgensen (2012). In addition, these bonds can be used by banks to satisfy liquidity regulation and (in the expanded version of the model) to deal with liquidity risk – two other new features in this paper. A fourth new element is that the model allows banks to be special through their lending activities as well as in liquidity provision, which could reflect banks' ability to monitor borrowers. Following Elenev et al. (2021) and Begenau and Landvoigt (2022), this is modelled through a productivity advantage of bank loans relative to loans extended by nonbank financial intermediaries, which compete with banks. The model and all main results are general in the sense that this productivity advantage may be positive, zero, or even negative.

Since a central goal of the model is to provide a framework not just for illustrating, but for actually measuring the welfare cost of liquidity and capital requirements, it is important to model the preferences for liquidity in a way that is not too restrictive. As much as possible, the data should be allowed to provide the answer, not special modeling choices. To that end, we follow a large literature in monetary economics (e.g., Sidrauski (1967), Lucas (2000), Woodford (2003)) in adopting the modeling device of putting liquidity services in the utility function, a method that is functionally equivalent to a range of more specialized, micro-founded models of liquidity demand (Feenstra (1986)). In that equivalence, the utility function emerges as a derived utility function. A growing literature in finance has applied this approach to a broader range of assets besides money, such as Treasuries (e.g., Krishnamurthy and Vissing-Jorgensen (2012)) and privately-created safe assets (Stein (2012), Hanson et al. (2015)), including bank deposits (Van den Heuvel (2008), Begenau (2020), Begenau and Landvoigt (2022)). The main advantage of this approach is its flexibility. Crucially, all main results will be derived without making any assumptions on the functional form of the utility function, beyond the standard assumptions that it is increasing and concave, thus allowing the data to speak.

The economy consists of households, banks, nonbank financial firms, nonfinancial firms, and a government. Households own the banks and firms. Nonfinancial firms produce output by employing labor and capital, which may be financed through bank or nonbank credit.

2.1 Households

There is a continuum of identical households with mass one. Households are infinitely lived dynasties and value consumption and liquidity services. They can obtain these liquidity services by allocating some of their wealth to deposits, an asset created by banks for this purpose. In addition, households also derive a convenience value from holding government bonds, reflecting their liquidity and safety. One can think of the household sector in this model as also encompassing money market funds, bond funds, or pension funds, which often manage households' holdings of government bonds on their behalf.

Besides holding bank deposits, denoted d_t, or government bonds, b_t, households can store their wealth by holding equity, e_t. They supply a fixed quantity of labor, normalized to one, for a wage, W_t. Taxes are lump-sum and equal to T_t. There is no aggregate uncertainty, so the representative household's problem is one of perfect foresight:

$$\max_{\{c_t,d_t,b_t,e_t\}_{t=0}^{\infty}} \sum\nolimits_{t=0}^{\infty} \beta^t u(c_t, d_t, b_t)$$

$$\text{s.t.} \quad d_{t+1} + b_{t+1} + e_{t+1} + c_t = W_t 1 + R_t^D d_t + R_t^B b_t + R_t^E e_t - T_t$$

and subject to a no-Ponzi-game condition and initial wealth constraint for $d_0 + b_0 + e_0$. c_t is consumption in period t, β is the subjective discount factor and R_t^D, R_t^B and R_t^E are the returns on bank deposits, government bonds, and equity, respectively. The returns and the wage are determined competitively, so the household takes these as given. There is no distinction between bank and other equity, since, in the absence of aggregate risk, they are perfect substitutes for the household and will thus yield the same return.

The utility function is assumed to be concave, at least once continuously differentiable on $\mathbb{R}^3_{++}$, increasing in all arguments, and strictly so in consumption: $u_c(c,d,b) \equiv \partial u(c,d,b)/\partial c > 0$, $u_d(c,d,b) \equiv \partial u(c,d,b)/\partial d \geq 0$ and $u_b(c,d,b) \equiv \partial u(c,d,b)/\partial b \geq 0$.

The first-order conditions to the household's problem are easily simplified to

$$R_t^E = \left(\beta u_c(c_t, d_t, b_t)/u_c(c_{t-1}, d_{t-1}, b_{t-1})\right)^{-1} \tag{1}$$

$$R_t^E - R_t^D = u_d(c_t, d_t, b_t)/u_c(c_t, d_t, b_t) \tag{2}$$

$$R_t^E - R_t^B = u_b(c_t, d_t, b_t)/u_c(c_t, d_t, b_t) \tag{3}$$

Equation (1), which determines the return on equity, is the standard intertemporal Euler equation for the consumption-saving choice, with one difference: the marginal utility of consumption may depend on deposits and bond holdings. Because there is no aggregate risk, the return on equity is essentially a risk-free rate on an asset that does not provide any liquidity benefits. Equation (2) captures the convenience yield on bank deposits. If $u_d > 0$, the interest rate on bank deposits is below the equity return reflecting the liquidity services provided by deposits. Equation (3) relates the spread between equity and bonds to the liquidity services of bonds in a similar fashion.

2.2 Banks

There is a continuum of banks, which make loans to nonfinancial firms, may hold government bonds, and finance these assets by accepting deposits from households and issuing equity. Banks last until they fail or choose to exit.[7] Banks' technology exhibits constant returns to scale and there is free entry into banking, so banks operate in an environment of perfect competition. The mass of banks is normalized to one. The balance sheet, and the notation, for the representative bank during period t is:

Assets		**Liabilities**	
L_t	Loans	D_t	Deposits
B_t	Bonds	E_t	Bank Equity

Loans yield a gross rate of return R_t^L at the end of the period t. R_t^L is determined competitively in equilibrium, so each bank takes it as given. In this baseline version of the model, loans are risk-free, while the expanded version of the model will consider loans with undiversifiable risk at the bank level. Similarly, the bank takes as given the return on the (riskless) government bonds, R_t^B, and the interest rate on (insured) deposits, R_t^D.

For quantitative realism, the model allows for resource costs associated with servicing deposits and/or making loans. Specifically, a bank incurs a noninterest cost $g(D, L)$ to service those financial contracts. g is assumed to be nonnegative, twice continuously differentiable on its domain $\mathbb{R}_+^2$, (weakly) increasing, convex and homogenous of degree 1, i.e. it exhibits constant returns to scale. Note that costless intermediation is included as a special case ($g \equiv 0$), as is a linear cost function.

Banks are subject to regulation by the government. First, banks face a capital requirement, which requires them to have a minimum amount of equity as a fraction of risk-weighted assets. In the context of this simple model, the capital requirement states that equity needs be at least a fraction γ of loans for a bank to be able to operate:

$$E_t \geq \gamma L_t$$

[7]Exiting takes the form of operating with scale set to zero.

For the moment, the capital requirement is merely assumed. Section 6 will show how it can be socially desirable to have such a requirement, as it mitigates the moral hazard problem that arises from deposit insurance or other government guarantees. There is no rationale in the economy for requiring equity against the bank's holdings of government bonds (which are assumed to be riskless). Accordingly, we have assumed that government bonds have a zero risk weight. This is also consistent with actual regulations.

Second, banks must satisfy a liquidity requirement by holding a minimum level government bonds, set equal to a fraction λ of deposits:

$$B_t \geq \lambda D_t$$

Again, for the moment this regulation is merely assumed, but section 6 will show how it can be socially desirable in the presence of liquidity risk and deposit insurance.

The bank maximizes shareholder value, net of the initial equity investment:[8]

$$\pi^B = \max_{L,B,D,E} \left[R^L L + R^B B - R^D D - g(D,L)\right] / R^E - E$$

$$\text{s.t.} \quad L + B = E + D, \quad E \geq \gamma L, \quad B \geq \lambda D \tag{4}$$

The constraints are, respectively, the balance sheet and the capital and liquidity requirements. The term $R^L L + R^B B - R^D D - g(D,L)$ is the bank's net cash flow at the end of the period. It consists of interest income from loans and bonds minus the interest owed to depositors, and minus the resource cost of intermediation. Shareholders receive this amount as dividends in return for their initial investment, E.[9] At the beginning of period t shareholders discount the value of end-of-period dividends by the opportunity cost of holding this particular bank's equity. This opportunity cost is R^E, the market return on equity.

It is straightforward to solve this problem (see Appendix A). To summarize the results, it is convenient to first define the all-in cost of financing a unit of loans with deposits, taking into account the liquidity requirement (but setting aside the transaction costs $g(D,L)$):

$$\tilde{R}^D(\lambda) \equiv R^D + \frac{\lambda}{1-\lambda}(R^D - R^B) \tag{5}$$

[8] Each bank is potentially long-lived. However, because there are no adjustment costs, nor any agency problems between banks and the other agents, its decision problem can be separated into a series of independent static decision problems without loss of generality. In what follows, time subscripts will be used only where necessary to avoid confusion.

[9] Here, we abstract from limited liability for shareholders. This is without loss of generality in the baseline model, as absent bank risk, dividends are always positive in equilibrium.

This reflects the fact that a fraction λ of the deposits must be invested in bonds, rather than loans, so to finance one unit of loans with deposits, $1/(1-\lambda)$ deposits must be raised, of which $\lambda/(1-\lambda)$ are put in bonds. If the return on bonds is less than the interest paid to depositors, then the liquidity requirement effectively increases the cost of financing loans with deposits. With that, the following proposition summarizes the solution.

Proposition 1 (solution to the bank's problem) *A finite solution requires*

$$R^B \leq R^D + g_D(D, L) \leq R^L - g_L(D, L) \leq R^E \tag{6}$$

The liquidity requirement binds if and only if the first inequality is strict. The capital requirement binds if and only if the last inequality is strict, or equivalently, if and only if $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L) < R^E$*. The solution satisfies the zero-profit condition:*

$$R^L - g_L(D, L) = \gamma R^E + (1-\gamma)\left\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L)\right\} \tag{7}$$

resulting in $\pi^B = 0$*. Finally, lemma 1 in the appendix enumerates when each of the two regulatory requirements is binding or slack as a function of variables that the bank takes as given only (that is, independent of* D *and* L*). All four cases are possible.*

Proof: See Appendix A.

Equation (7) has the interpretation of a zero-profit condition. For intuition, consider the special case of costless intermediation ($g \equiv 0$), for which the condition simplifies to

$$R^L = \gamma R^E + (1-\gamma)\tilde{R}^D(\lambda)$$

With a binding capital requirement, one unit of lending is financed by γ in equity and $(1-\gamma)$ in deposits. Thus, competition will equalize the lending rate to the similarly weighted average of the required rates of return of equity and deposits, using the all-in cost for deposits, $\tilde{R}^D(\lambda)$, to account for the liquidity requirement's prescription that some of the deposits raised be invested in government bonds.[10] For the general case with positive noninterest costs, the condition is identical, except that the marginal cost of screening and servicing loans ($g_L(D, L)$) is deducted from the lending return, and the cost of deposit finance now includes not only the interest costs, but also the cost of servicing an additional

[10] The condition holds with a nonbinding capital requirement as well, since $R^E = \tilde{R}^D(\lambda)$ in that case, as shown in the proposition (using $g = 0$).

unit of deposits, $g_D(D,L)$.[11]

The liquidity requirement binds whenever investing in government bonds and financing those with deposits is a money-losing strategy; that is, whenever $R^B < R^D + g_D(D,L)$. Under that condition, it is straightforward to show that $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D,L)$ exceeds $R^D + g_D(D,L)$, so that a binding liquidity requirement raises the total (i.e., all-in interest plus noninterest) cost of funding loans with deposits.

The capital requirement binds if equity finance is more expensive than deposit finance, taking into account the impact of the liquidity requirement on the all-in cost of deposit finance and the noninterest cost of servicing deposits. In that situation, the rate on loans will be strictly in between R^E and $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D,L)$. Regardless of whether the two regulatory constraints are slack or binding – all four cases are possible – economic profits are zero due to the constant returns to scale and perfect competition. Shareholders simply get the competitive return, R^E.

2.3 Firms

Nonfinancial firms cannot create liquidity though deposits. They can, however, produce the economy's single good by hiring labor and using physical capital. Capital (K_t) is purchased at the beginning of the period and is financed with a combination of bank and nonbank loans: $K_t = L_t + L_t^{NB}$ (nonbanks are discussed next).[12] Since these funding sources are perfect substitutes for the firm, their cost is the same, R_t^L.[13] Technology is standard. Output in period t is $F(K_t, H_t)$, where H_t is hours of labor input and $F()$ is a well-behaved production function exhibiting constant returns to scale. A fraction δ of the capital stock depreciates during the period. Firms last for one period[14] and each period, there is a continuum of firms with mass normalized to one, so each firm takes prices as given. The firm maximizes profits:

$$\pi_t^F = \max_{K_t, H_t} F(K_t, H_t) + (1-\delta)K_t - W_t H_t - R_t^L K_t$$

[11] To finance one unit of loans $1/(1-\lambda)$ units of deposits must be raised due to the liquidity requirement, whence the presence of that factor in (7).

[12] It is assumed that firms do not issue equity to households. Explicit consideration of this possibility would not change the results, provided firms' equity issuance costs are equal to or larger than nonbanks' intermediation cost, the parameter φ introduced below. Note that the case $\varphi = 0$ is allowed. Similarly, in the model, nonfinancial firms do not want to hold any government bonds as $R^B \leq R^L$ in equilibrium, so explicit consideration of the possibility would not change any of the results.

[13] As mentioned, the model allows for a productivity advantage of bank loans. This is captured through the intermediation cost of nonbanks in the next subsection.

[14] The absence of adjustment costs and agency problems implies that this is without loss of generality. One can think of ongoing firms as repurchasing their capital stock each period.

yielding standard first-order conditions for the choices of labor and capital:

$$(H_t) \quad F_H(K_t, H_t) = W_t \tag{8}$$

$$(K_t) \quad F_K(K_t, H_t) + 1 - \delta = R_t^L \tag{9}$$

These optimality conditions, together with the constant returns to scale assumption, imply that profits, π_t^F, equal zero.

2.4 Nonbank Financial Intermediaries

Nonbank financial intermediaries (NBFIs) compete with banks in making loans to firms. Unlike banks, they do not have access to the deposit technology. Thus, NBFI loans, L_t^{NB}, are funded by equity issued to households, E_t^{NB}. [15] Without loss of generality, NBFIs last for one period, and there is a continuum with mass one, so they are price takers.

Just as for banks, we allow for noninterest costs of originating loans by NBFIs, equal to $\varphi \geq 0$ per unit lent. Note that costless intermediation is allowed as a special case. Conversely, a high value of φ (one that exceeds the bank's cost, g_L) can capture the notion that banks have a special advantage in providing credit to firms, a modelling approach that follows Elenev at al. (2021) and Begenau and Landvoigt (2022).

Since NBFI can make loans at constant marginal cost given by $R_t^E + \varphi$, the equilibrium lending rate has an upper bound:

$$R_t^L \leq R_t^E + \varphi, \text{ with equality if } L_t^{NB} > 0 \tag{10}$$

A higher loan rate would imply infinite profits and scale for NBFIs, which is inconsistent with equilibrium. If $R_t^L < R_t^E + \varphi$, NBFIs cannot compete with banks, in which case banks are the only type of lender. Reflecting constant returns to scale, NBFI economic profits equal zero even if they do operate, and their shareholders receive the market return, R_t^E.

2.5 Government

The government's fiscal policy is to maintain a constant level of government debt, $\bar{B}$. Lump-sum taxes are

$$T_t = (R_t^B - 1)\bar{B} \tag{11}$$

[15] This is without loss of generality in the sense that NBFIs could equally rely on debt funding from households, as long as their debt does not provide liquidity services.

The government also sets the capital and liquidity requirements. For now, their values are taken as parametric, with a discussion of optimal policy deferred to section 6.

3 General Equilibrium

This section presents a positive analysis of general equilibrium. Given a government policy λ and γ, an equilibrium is defined as a path of consumption, capital, employment, and financial quantities and returns, for $t = 0, 1, 2, \dots$, such that households, banks and firms all solve their maximization problems, taxes are set according to (11), and all markets clear:

$$e_t = E_t + E_t^{NB}, \quad d_t = D_t, \quad L_t + L_t^{NB} = K_t, \quad B_t + b_t = \bar{B}, \quad H_t = 1 \tag{12}$$

and, for the goods market,

$$F(K_t, 1) + (1 - \delta)K_t = c_t + K_{t+1} + g(D_t, L_t) + \varphi L_t^{NB} \tag{13}$$

By combining these market clearing conditions, equations (1), (2), (3), (5), (8), (9) and (10), and proposition 1, the equilibrium allocation can be characterized as a dynamic system in (K_t, c_t). This system is shown in full in Appendix B, which also provides a more technical discussion of its characteristics than what follows. Here, we highlight some key features of the equilibrium.

First, *the bank capital requirement typically binds* in equilibrium due to the convenience yield on deposits, which makes them a cheaper source of funds for banks than equity (see (2)). For example, without noninterest costs of banking ($g = 0$), the capital requirement binds whenever the convenience yield on deposits exceeds a fraction λ of the convenience yield on government bonds.[16]

Second, *the liquidity requirement may or may not bind, depending on the convenience yield of government bonds relative to bank deposits.* Specifically, it binds when, at the margin, the convenience yield of government bonds exceeds the convenience yield of bank deposits, net of the marginal noninterest costs of servicing those deposits; that is, if

$$u_b(c_t, d_t, b_t) > u_d(c_t, d_t, b_t) - g_D(d_t, L_t)u_c(c_t, d_t, b_t)$$

[16] With positive noninterest costs, the capital requirement binds under the same condition provided the convenience yield on deposits is taken net of their marginal noninterest costs; that is, if $u_d(c_t, d_t, b_t) - g_D(d_t, L_t)u_c(c_t, d_t, b_t) > \lambda u_b(c_t, d_t, b_t)$. Moreover, as shown in Appendix B, this condition always holds in equilibrium if $\varphi < g_L(d_t, L_t)$.

Third, *investment can be affected by the capital requirement as well as the liquidity requirement, if binding.* In equilibrium, the marginal product of capital is equated with the lending rate, which can be lower than the cost of equity and depend on these requirements. Two features of the model are key to understanding how and when this can happen. First, as noted, households' liquidity preference implies that the pecuniary return on deposits is lower than the return on equity. Second, competitive banks will pass on the cheap deposit finance in the form of a lower lending rate, but this pass-through is moderated by regulation.

Take the case that the capital and liquidity requirements both bind. Then the cheap deposit finance lowers the bank lending rate by $(1-\gamma)(R^E - \tilde{R}^D(\lambda))$, as a fraction γ of loans is still financed with bank equity and as the liquidity regulation raises the all-in cost of financing loans with deposits by $\tilde{R}^D(\lambda) - R^D = \frac{\lambda}{1-\lambda}(R^D - R^B)$. Using the households' first-order conditions for deposits and bonds and taking into account noninterest costs yields a net reduction in the lending rate, relative to the return on equity, that is equal to

$$R_t^E - R_t^L = \Delta_{L,t} \equiv \frac{1-\gamma}{1-\lambda}\left(\frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - g_D(d_t,L_t) - \lambda\frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)}\right) - g_L(d_t,L_t) \quad (14)$$

This is the equilibrium analogue to the bank's zero profit condition (7). Thus, the marginal product of capital is given by

$$F_K(K_t,1) + 1 - \delta = R_t^L = \beta^{-1}\frac{u_{c,t-1}}{u_{c,t}} - \Delta_{L,t} \quad (15)$$

(see (1) and (9)). If the spread Δ_L is positive, or at least strictly greater than $-\varphi$, so that $R^L < R^E + \varphi$, equity-financed NBFIs cannot compete with banks, and firms will rely exclusively on the cheaper bank loans to finance investment (see (10)). In such a **'pure bank finance'** equilibrium, $L = K$.[17] In that equilibrium, because banks pass on the low cost of deposits to firms, the steady state capital stock will be higher than the level implied by the standard growth model's modified golden rule if $\Delta_L > 0$.

Moreover, as a consequence, the steady state levels of the capital stock and income per capita are not invariant to changes in the liquidity requirement or in the capital requirement, as these requirements influence the spread $R^E - R^L$; see (14). With respect to the capital requirement, this non-invariance result is similar to the one obtained in Van den Heuvel (2008) and explored more fully in Begenau (2020) and Van den Heuvel (2006). With respect to the liquidity requirement, the non-invariance result is, as far as we know, novel within

[17] Appendix B shows that the condition $\Delta_L > -\varphi$ is a necessary and sufficient condition for a pure bank finance equilibrium ($L = K$), regardless of the bindingness of the regulatory requirements.

the context of this type of model.[18]

As a fourth key feature, *the equilibrium can also be characterized by* **'mixed finance,'** where firms finance investment with a combination of bank and nonbank funding (so $L < K$). In the model, nonbank funding takes the form of loans from NBFIs, but this can be interpreted more broadly as including funds raised on capital markets or from shadow banks. Technically, the mixed finance equilibrium occurs if $\Delta_L < -\varphi$ when evaluated at $L = K$, where φ parameterizes the specialness of bank loans. Thus, loosely speaking, if bank monitoring adds little value, a mixed equilibrium is more likely. In such an equilibrium, firms use both bank and NBFI loans, in such proportion that their rates are equal (thus, $R^L = R^E + \varphi$ or, equivalently, $\Delta_L + \varphi = 0$), which endogenously determines the relative size of the bank and nonbank sectors.[19]

Fifth, *very stringent regulation can lead to a shift to nonbanks, shadow banks, or disintermediation.* Intuitively, the mixed finance equilibrium prevails when the resource cost of bank intermediation, g, is high relative to the resource cost of NBFI loans, φ, when the liquidity value of deposits is low or the capital requirement high, or because of the combination of a high liquidity requirement, λ, and a high liquidity premium (low yield) on government bonds; see (14). Thus, a very high capital or liquidity requirement can cause migration toward nonbank finance, whether through capital markets or through nonbank intermediaries, such as shadow banks. For the liquidity requirement, this is more likely to happen if the convenience yield on government bonds is high; that is, when the supply of government bonds or close substitutes is low relative to the demand for such assets. Of course, shadow banking can bring its own financial stability risks, such as run risks (e.g., Begenau and Landvoigt (2022)), though Ordoñez (2018) argues that it can also provide a socially beneficial channel to escape excessive regulation.

Figure 1 illustrates the effects of tighter regulation. It traces, for a data-driven para-

[18] The result contrasts starkly with the well-known superneutrality result of the Sidrauski (1967) model. In that model, liquidity preference and monetary policy do not influence the steady state capital stock. A key difference is that the supply of liquid assets (money) is exogenous in the Sidrauski model.

[19] In contrast to the pure bank finance case, with $R^L = R^E + \varphi$ in a mixed finance equilibrium, the steady state level of the capital stock satisfies $MPK = \beta^{-1} + \varphi$. Accordingly, the long-run level of capital is independent of liquidity preference or any banking or regulatory parameters, although these elements do influence the composition of the financial sector, as explained below. Moreover, absent the friction φ, the implied level of the capital stock satisfies the modified golden rule in this case.

Given that firms in the real world do not exclusively use bank loans, it may seem that the mixed finance equilibrium is more realistic, and that the dependence of economic activity in the long run on regulation is a mere theoretical possibility. However, that would be taking the model too literally in my view, as, in reality, bank and nonbank funding are not perfect substitutes for all firms, as they are in the model by simplifying assumption. In reality, some firms are bank-dependent, and even firms that can access capital markets often rely on backup lines of credit from banks to facilitate that access.

Figure 1: Economic impact of large increases in the liquidity and capital requirements

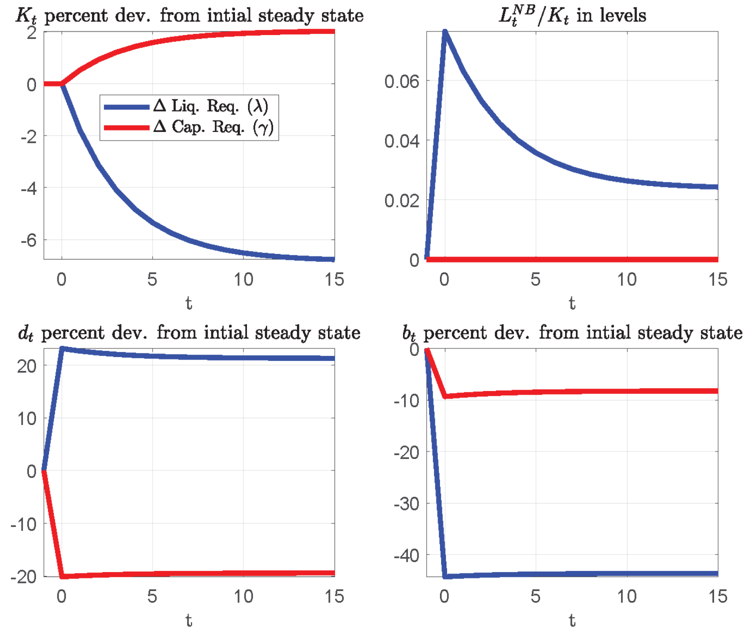


Illustration of the effects of a 25 p.p. unanticipated and permanent increase in the liquidity or capital requirement. Parameterization from Table D.3, column (1). See Appendix D for details.

meterization of the economy,[20] the dynamic effects of a large increase in the liquidity requirement (in blue) and the capital requirement (red). As shown in the top left, investment drops significantly as a result of the higher liquidity requirement, resulting in a temporary consumption boom, while there is a modest increase in investment from a tighter capital requirement.[21] For the liquidity requirement, a higher bank lending rate leads to entry by nonbanks (top right), and thus a shift from pure bank finance to mixed finance. The bottom two panels show the effects on liquidity services. With a higher liquidity requirement, banks must hold more government bonds, so household holdings fall considerably. However, deposits move in the opposite direction by a similar dollar amount, as banks finance their bond holdings with deposits. The net impact on liquidity provision is small. In contrast, with a higher capital requirement, deposits shrink significantly, and there is no offsetting effect of government bonds, resulting in a larger impact on liquidity provision.

[20] As explained in Appendix D, the figure's calibration is based on the pre-Basel III period and an initial steady state with pure bank finance to illustrate a transition from pure bank to mixed finance.

[21] The latter result may seem surprising, but it is consistent with Begenau (2020) and Van den Heuvel (2006), who show that this outcome occurs when the interest-elasticity of the demand for deposits is low, as it is in this illustration. In that case, the decline in deposits from an increase in the capital requirement leads to a large fall in the equilibrium deposit rate, which in turn reduces banks' lending rate.

4 The Welfare Costs of Regulation

To quantify the welfare cost of the liquidity and capital requirements, we will use a social planner's problem that is constrained to respect the regulations and designed to replicate the decentralized equilibrium, rather than to solve for the first-best. The equivalence between planner and equilibrium allocations will then be exploited to analytically derive two simple formulas that will serve as sufficient statistics for the welfare costs of the requirements.

Define the following constrained social planner's problem:

$$\begin{aligned} V_0(\theta) \quad = \quad & \max_{\{c_t,d_t,b_t,B_t,L_t,K_{t+1}\}_{t=0}^{\infty}} \sum\nolimits_{t=0}^{\infty} \beta^t u(c_t, d_t, b_t) \\ & \text{s.t.} \quad K_{t+1} = F(K_t, 1) + (1-\delta)K_t - c_t - g(d_t, L_t) - \varphi(K_t - L_t), \\ & \qquad B_t \geq \lambda d_t, \ (1-\gamma)L_t + B_t \geq d_t, \ B_t + b_t = \bar{B}, \ K_t \geq L_t \end{aligned} \tag{16}$$

where $\theta = (\lambda, \gamma, K_0)$. The constraints correspond to the social resource constraint, the liquidity requirement, the capital requirement, bond market clearing, and the nonnegativity constraint on NBFI lending, in that order. Appendix B shows that the allocation of this planner is identical to the decentralized equilibrium. Therefore, the planner's problem replicates the decentralized equilibrium, and welfare in that equilibrium equals $V_0(\theta)$.

With that, it is straightforward to derive expressions for the marginal welfare costs of the two regulations by differentiating $V_0(\theta)$ with respect to λ and γ, using the envelope theorem. Exploiting the equivalence of the planner's allocation with the decentralized equilibrium's and using the households' optimality conditions for the choices of deposits and bonds, (2) and (3), yields the two formulas for the gross marginal welfare costs. These are presented in the next two propositions, starting with the liquidity requirement:

Proposition 2 (welfare cost of the liquidity requirement) *Assume that the economy is in steady state in the current period. The marginal welfare cost of the liquidity requirement, expressed as the welfare-equivalent permanent relative loss in consumption, equals*

$$\nu_{Liq} = \frac{d}{c} \left(R^D + g_D(d, L) - R^B \right) (1 - \lambda)^{-1} \tag{17}$$

Thus, a first-order approximation of the welfare cost of permanently increasing the liquidity requirement λ by $\Delta\lambda$ is $\nu_{Liq}\Delta\lambda$.

Proof: See Appendix B.

The above formula is empirically implementable. Remarkably, it does not rely on any assumptions about the functional form of preferences, beyond the standard assumptions of monotonicity, differentiability and concavity. Instead, the formula relies on asset yields to reveal the strength of households' preferences for liquidity. In addition, the measurements presented below will also avoid making any functional form assumptions on the cost function g. As is common for "sufficient statistics" formulas, there are multiple combinations of primitive parameters and functional forms that are consistent with the inputs to the formulas, and all such combinations have the same welfare implications (Chetty, 2009).

The result shows that there is a positive (gross) welfare cost associated with bank liquidity regulation only to the extent that the interest rate on deposits, plus the marginal cost of servicing deposits, exceeds the interest rate on government bonds. The logic is simple: from the perspective of the other agents, the liquidity requirement effectively forces banks to transform some government bonds into deposits, both instruments prized for their liquidity. Thus, imposing a liquidity requirement entails a social cost only to the extent that the liquidity services of deposits are, at the margin and net of the noninterest cost of creating these services, valued less than those of Treasuries; only then is there a costly *net* reduction in liquidity available to investors. The deposit-Treasury spread, adjusted for the noninterest cost of deposits, reveals whether this is true or not.

Importantly, the formula takes into account gains and losses associated with the move to a new steady state. While the expression in (17) assumes that the economy is in steady state in the *current* period, equation (50) in the Appendix gives the result without that assumption, and it suggests that the sufficient statistic in (17) should provide a reasonable approximation even outside a current steady state.

Further, the formula is valid whether the equilibrium is characterized by pure bank finance or by mixed bank and nonbank finance, and even if the liquidity requirement does not bind (in which case $R^D + g_D(d, L) = R^B$, so $\nu_{Liq} = 0$ as expected). The regulation entails a gross social cost whenever the requirement binds and is costless otherwise.

Finally, the formula applies whether bank loans are special or not. Recall that depending on the level of NBFI intermediation costs (φ), bank loans may or may not have a productivity advantage. Proposition 2 applies regardless. This may seem surprising, but it is important to keep in mind that the equilibrium values of the deposit-consumption ratio and the cost-revealing spread in the formula generally depend on the specialness of bank loans. Specifically, in a mixed finance equilibrium (that is, if NBFI lending occurs), a rise in φ will reduce nonbank credit, increase bank loans and thus deposits (assuming the capital requirement binds). In turn, this will increase the deposit-consumption ratio. In addition, as deposits become less scarce and banks must hold more of the government bonds, it will tend to raise the equilibrium spread between deposits and those bonds.[22] Both effects will result in a higher marginal welfare cost of the liquidity requirement. Thus, bank regulation may be more costly if banks have innate advantages in lending as well as in liquidity creation. The good news is that we will still be able to measure that cost in the same way.

The next key proposition presents a formula for the marginal gross welfare cost of the capital requirement:

Proposition 3 (welfare cost of the capital requirement) *Assume that the economy is in steady state in the current period. The marginal welfare cost of the capital requirement, expressed as the welfare-equivalent permanent relative loss in consumption, equals*

$$\nu_{Cap} = \frac{L}{c}\left(R^E - \tilde{R}^D(\lambda) - (1-\lambda)^{-1} g_D(d, L)\right) \tag{18}$$

Thus, a first-order approximation of the welfare cost of permanently increasing the capital requirement γ by $\Delta\gamma$ is $\nu_{Cap}\Delta\gamma$.

Proof: See Appendix B.

Recall that $\tilde{R}^D(\lambda) \equiv R^D + \frac{\lambda}{1-\lambda}(R^D - R^B)$. Again, the above formula is empirically implementable, does not rely on any assumptions about the functional form of preferences, is valid whether the equilibrium is characterized by pure bank finance or by mixed finance, and even if the capital requirement does not bind (in which case $R^E = \tilde{R}^D(\lambda) + (1-\lambda)^{-1} g_D$, so $\nu_{Cap} = 0$ as expected). Again, also, it applies whether bank loans are special or not, as the equilibrium ratio of bank loans to consumption and the cost-revealing spread adjust to the specialness of bank loans. And again, it takes into account gains and losses associated with the move to a new steady state. While it assumes that the economy is in steady state in the *current* period, equation (52) in the Appendix provides the marginal welfare cost without that assumption, suggesting that the expression in (18) should be a reasonable approximation even outside a steady state.

An increase in the capital requirement entails a welfare cost because it constrains the ability of banks to issue deposit-type liabilities, which are valued by households for their liquidity. The spread between the risk-adjusted[23] return on bank equity and the pecuniary return on deposits, $R^E - R^D$, reveals the strength of households' preferences for the liquidity services of deposits. However, the production of these services also entails noninterest costs,

[22] The rise in the spread may not occur if deposits and Treasuries are complements for households.

[23] Recall that bank equity is aggregate-risk-free in the model, so there are no Modigliani-Miller 'offsets.' However, when the model is confronted with the data in the next section, this will be a key area of concern.

$g_D(d, L)$, and requires banks to hold more government bonds –which are also prized for their liquidity– to satisfy the liquidity requirement. To account for this, the formula deducts the marginal noninterest cost of deposits[24] and factors in the impact of the liquidity requirement, λ, by using the all-in cost of financing loans with deposits, $\tilde{R}^D(\lambda)$, instead of R^D. Only if a positive spread remains after these adjustment, is a scarcity of deposits due the capital requirement revealed and only then is there a welfare effect at the margin.

This result generalizes the one in Van den Heuvel (2008), which does not feature liquidity regulation. Proposition 1 in the latter paper is nested by setting $\lambda = 0$ in (18):[25]

$$\nu_{Cap}|_{\lambda=0} = (L/c)\left(R^E - R^D - g_D(d, L)\right) \tag{19}$$

Relatedly, the impact of the liquidity requirement on the welfare cost of the capital requirement can be seen more explicitly by rewriting (18) using (17) and (5):

$$\nu_{Cap} = \nu_{Cap}|_{\lambda=0} - (L/d)\lambda\nu_{Liq} \tag{20}$$

Thus, *for given observables* (R^E, R^D, etc.) imposing a liquidity requirement lowers the welfare cost of the capital requirement if the liquidity requirement binds. Of course, these observables are generally not completely invariant to changes in the liquidity requirement.

5 Measurement of the Welfare Costs

The goal of this section is to measure the gross welfare costs of bank liquidity requirements and capital requirements by combining the formulas derived in the previous section with data. To that end, we use annual aggregate balance sheet and income statement data for all FDIC-insured commercial banks in the United States. These data are obtained from the FDIC's Historical Statistics on Banking (HSOB) and are based on regulatory filings ('call reports'). We employ data from the period 1986 to 2019.[26]

A key challenge to the empirical application of the formulas in (17) and (18) is the measurement of the marginal net noninterest cost of servicing deposits, g_D. This includes the cost of ATMs, some of the cost of maintaining a network of branches, etc. Fees on

[24] The factor $1/(1-\lambda)$ multiplying $g_D(d, L)$ reflects the fact that the bank must raise $1/(1-\lambda)$ in deposits to finance one unit of lending, while satisfying the liquidity requirement.

[25] The statement of proposition 1 in Van den Heuvel (2008) also uses the fact that, in that model, $L = d/(1-\gamma)$ if $R^E - R^D - g_D(d, L) \neq 0$.

[26] Regulation Q, which placed restrictions on banks' deposit rates, was fully phased out on January 1, 1986.

deposits should be netted out. The call reports contain data on noninterest expense and revenue, and the difference, net noninterest cost, is nontrivial, averaging 1.3 percent of total assets on an annual basis over the 1986-2019 period. However, there is little information in the data permitting a breakdown by activity (e.g. servicing deposits, screening loan applications, collecting payments, etc.).

Fortunately, however, the model suggests a way to infer the marginal net noninterest cost of deposits when banks voluntarily hold Treasuries on their balance sheet. Specifically, proposition 1 shows that whenever the liquidity requirement is not binding, then

$$R^D + g_D(d, L) = R^B \tag{21}$$

The interpretation is banks will only hold more Treasuries than required if investing in Treasuries and financing them with deposit-type liabilities is not a money-losing activity, taking into account noninterest costs. Thus, by finding a historical period when banks held Treasuries well in excess of any regulatory requirements, we can infer g_D from that period's Treasury-deposit spread.

Figure 2 shows U.S. Treasuries and excess reserves held by U.S. depository institutions, expressed as a share of total assets, from 1986 to 2019.[27] As can be seen from the chart, between 1986 and 2000, banks invested a significant part of their balance sheet in Treasuries (more than 1 percent of total assets). This asset allocation was voluntary, as there was no Basel-style liquidity requirement applicable during this period (so $\lambda = 0$ in the sense of the model).[28] Reserve requirements were in place, but these could only be satisfied by holding reserves at the Fed, not by holding Treasuries. Thus, we use data from the 1986-2000 period to infer g_D using equation (21).[29]

For R^B, we use the 3-month Treasury bill rate on the secondary market. The average net interest rate on deposits, $R^D - 1$, is calculated as the HSOB's Interest on Total Deposits divided by Total Deposits.[30] For the period 1986-2000, the resulting average

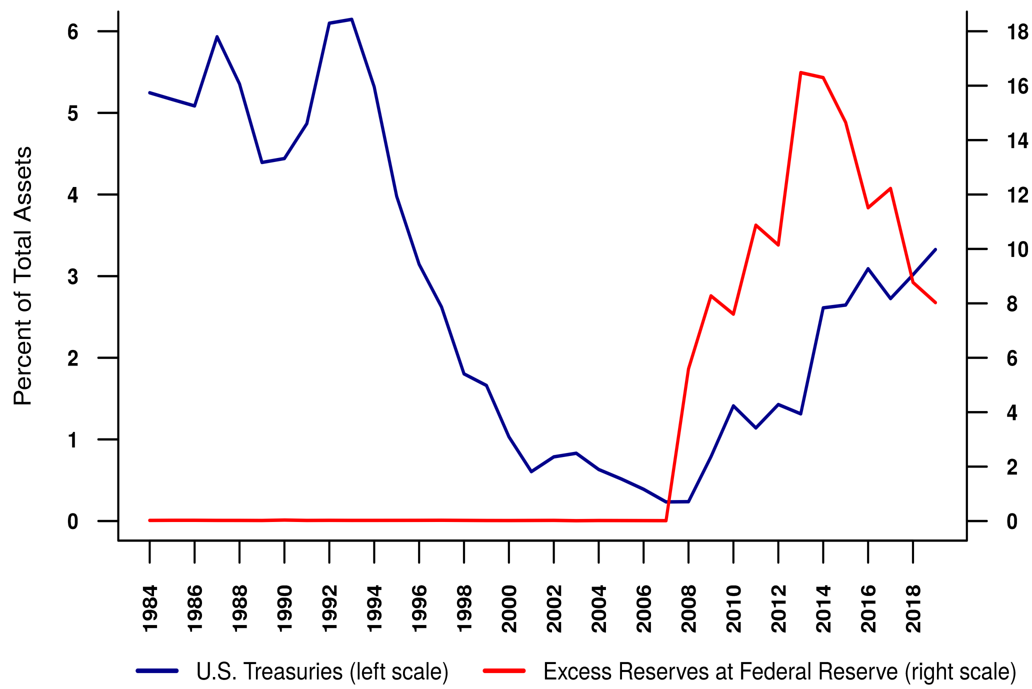
[27] In a sense, excess reserves can be thought of as holding Treasuries through the Federal Reserve's balance sheet. Before 2008, excess reserves were not remunerated by the Fed and were a negligible share of banks' total assets. In 2008, the Fed started paying interest on excess reserves and embarked on large scale asset purchases, developments that boosted excess reserves in the following years. See Ennis and Wolman (2015) for an empirical analysis of banks' excess reserves in this period.

[28] One could view this period as one where liquidity regulation was not needed because high-quality liquid assets were abundant so banks voluntarily held them in sufficient amounts.

[29] Starting around 2014, banks also held notable amount of Treasuries. However, as discussed in more detail below, holdings in this period are affected by the anticipation and implementation of liquidity regulation.

[30] All data are nominal. While the model is real, using nominal data consistently is correct, because the formulas for the welfare costs contain only ratios of quantities and spreads of returns.

Figure 2: U.S. Treasuries and excess reserves held by U.S. depository institutions



Note: Prior to 1991 Treasuries are for FDIC-insured commercial banks only and are sourced from Call Reports.
Source: FDIC Historical Statistics on Banking, Federal Reserve H.3 Release, and FFIEC Call Report.

Treasury-deposit spread, $R^B - R^D$, equals 1.22 percent. Accordingly, we set the marginal net noninterest cost associated with deposits at $g_D = 0.0122$ per annum.

For robustness, two additional estimates for the marginal net noninterest cost of deposits are obtained or derived from the literature.

1. Hanson, Schleifer, Stein, and Vishny (2015) use a hedonic regression approach and estimate the average cost of servicing deposits at 1.30 percent. Netting out the noninterest income from service charges on deposit accounts (0.49%) yields a marginal net noninterest cost of 0.81 percent, a little below the first estimate.

2. Corbae and D'Erasmo's (2021) estimate the marginal net noninterest cost of lending at 1.13 percent.[31] With the linear homogeneity of net noninterest cost in our framework, this implies a marginal net noninterest cost of deposits of 0.90 percent, in between the previous two estimates.[32]

[31] See table A.IV in Corbae and D'Erasmo (2021), third column, asset-weighted average for all banks.

[32] Linear homogeneity of $g(D, L)$ implies $g_D(D, L) = g(D, L)/D - (L/D)g_L(D, L)$. We set $g_L = 0.0113$

It is useful to compare these estimates to an upper bound that can be obtained by attributing all net noninterest cost to servicing deposits and none to lending. Maintaining the assumption of constant returns to scale of g, this upper bound equals $g_D(D, L) = g(D, L)/D$.[33] The latter ratio is equal to 0.0216 per annum, on average for the same time period. Consistent with the model, the upper bound exceeds the spread-based estimate and suggests that about half of total net noninterest cost can be attributed to deposits (slightly more than half for the spread-based estimate and a bit less than half for the other two estimates). This implication strikes us as plausible.

To map the data into the remaining variables, we largely follow Van den Heuvel (2008). For deposits, D, the HSOB's Total Deposits is used. For consumption, c, personal consumption expenditures from the NIPA is used. For loans, we use Total Assets net of U.S. Treasuries and excess reserves. To quantify the welfare costs, we calculate long run averages of the ratios and the spreads in the formulas in propositions 2 and 3, starting with the liquidity requirement.

5.1 Liquidity Regulation

We use data from two distinct periods to gauge the long-run economic costs of liquidity regulation. The first period is 2001-2006, a time without liquidity regulation and when the introduction of such regulation likely would have been binding. As can be seen in figure 2, in each of these years, banks' holding of Treasuries plus excess reserves were less than 1 percent of total assets, indicating that even a modest liquidity requirement would have necessitated changes to banks' balance sheets and making this period a good candidate to gauge its potential welfare cost. The second period, 2016-2019, covers the time after the implementation of the LCR, a key Basel III liquidity requirement.[34]

Starting with the pre-Basel III period, over 2001-2006, the average nominal yields on Treasuries and deposits are, respectively, 2.54% and 1.90%, so the average spread is 64 basis points, less than the marginal noninterest cost of servicing deposits, which we have already estimated at 122 basis points. The mean deposit to consumption ratio is 0.66, and, as already noted, there was no liquidity requirement in place ($\lambda = 0$). Applying (17), the first-order approximation to the gross welfare cost of introducing a liquidity requirement

and use our data for the same sample period as Corbae and D'Erasmo (1984-2016) to calculate the sample averages for the ratios g/D and L/D.

[33] Constant returns to scale imply $g(D, L) = g_D(D, L)D + g_L(D, L)L \geq g_D(D, L)D$.

[34] The years between the two periods contained the global financial crisis, three rounds of large scale asset purchases (QE) by the Fed, and several announcements that provided increasing clarity about the Basel III liquidity rules, making those years less arguably attractive for measuring their steady state welfare costs.

set at $\lambda_{new} > 0$ is:

$$\begin{aligned}\nu_{Liq}\lambda_{new} &= \frac{d}{c}\left(R^D + g_D(d,L) - R^B\right)(1-\lambda)^{-1}\lambda_{new} \\ &= 0.66 \times (0.0190 + 0.0122 - 0.0254) \times 1 \times \lambda_{new} = 0.0038 \times \lambda_{new}\end{aligned}$$

To interpret this number, consider the cost of a 10 percent liquidity requirement, a level that is roughly comparable to Basel III's LCR requirement.[35] Its gross welfare cost is equivalent to a permanent loss in consumption of

$$\nu_{Liq} \times 0.1 = 0.0038 \times 0.1 \times 100\% = 0.038\%$$

or about \$5.5 billion per year (using 2019 consumption). While perhaps not trivial, this is a relatively small number compared to estimates of the welfare cost of inflation (e.g., Lucas (2000)), or compared to many existing estimates of the cost of capital requirements (e.g., BCBS (2010)). Even modest financial stability benefits would easily justify this cost.[36]

Using the alternative estimates for the net noninterest cost of servicing deposits results in even lower measurements of the marginal welfare cost: $\nu_{Liq} = 0.001$ or 0.002 based on the estimates derived from, respectively, Hanson et al. (2015) ($g_D = 0.81$ percent) and Corbae and D'Ersamo (2021) ($g_D = 0.90$ percent). Using this, the gross welfare cost of a 10 percent liquidity requirement is equivalent to a permanent loss in consumption of only 1 or 2 basis points, a very small effect.

Our second set of measurements covers the period following the implementation of the LCR, 2016-2019.[37] Admittedly, this time span includes fewer years, raising the risk that short-term fluctuations in spreads have an outsize influence on the results. With that said, during this period, the average nominal yields on Treasuries and deposits are, respectively, 1.31% and 0.50%, so the average spread is 81 basis points, actually quite similar to the 2000s. The mean deposit to consumption ratio is 0.94, a somewhat higher value. To obtain a value for λ, we use the period's average ratio of depository institutions' holdings of Treasuries

[35] The LCR requirement depends on more detailed balance sheet information than can be captured in a macroeconomic model. However, internationally, 10% appears a reasonable ratio to capture the LCR. In the U.S., the ratio appears somewhat higher, as elaborated below.

[36] Note that the cost is nonnegative, as predicted by the model. Nothing in the empirical methodology guaranteed a nonnegative number, so this might be viewed as a small empirical validation of the model.

[37] The LCR and the modified LCR, a similar but less stringent requirement for smaller banks, were phased in at 90 percent of their final values beginning on Jan. 1, 2016 and at 100 percent beginning on Jan.1, 2017. Starting the measurement period in 2017 makes little difference to the results. Similarly, extending the sample to 2020, a year affected by the onset of the pandemic, has little impact.

plus excess reserves to their total deposits. This results in $\lambda = 0.17$.[38]

Combining these measurements with the first estimate of the net noninterest cost of servicing deposits (1.22%), yields a gross marginal welfare cost of the liquidity requirement in the post-implementation period equal to $\nu_{Liq} = 0.0046$. Again, to interpret this number, we can consider the gross welfare cost of a 10 percentage point increase in the liquidity requirement. To a first-order approximation, this cost is equivalent to a permanent loss in consumption of 0.046%, or about $6.6 billion per year.[39] Though about 20 percent higher than its pre-implementation value, it may still be considered a small welfare cost. Using the alternative estimates for the net noninterest cost of deposits again results in a lower marginal welfare cost: about 1 basis point or less of consumption for a 10 percentage point increase in the liquidity requirement.

5.2 Capital Regulation

To measure the welfare cost of capital requirements, we need an estimate of the required return on bank equity. Whereas the model abstracts from aggregate risk, a *risk-adjusted* measure is in fact called for. A risk adjustment captures the degree to which equity's required return adjusts in response to changes in leverage, such as those brought about by changes to capital requirements. In particular, for given asset risk, a decline in leverage should make bank shares less risky, and in theory this should lower the required return that shareholders demand. Indeed, under the idealized conditions underlying Modigliani and Miller's propositions, the strength of this effect is just such that the weighted average cost of funds does not depend on its leverage at all, so that the firm can change its leverage at zero cost.[40]

In reality, there are several reasons why the Modigliani-Miller theorem does not hold – agency problems, taxes, bankruptcy costs, etc. – and it is especially unlikely to hold for banks in light of the special nature of their debt. Indeed, in the model presented, the liquidity of bank debt is simultaneously a reason that banks exist and the reason the

---

[38] Because, as mentioned, the actual the LCR requirement depends on more detailed balance sheet information than can be captured in a tractable model, we measure its implied value for λ under the assumption that the LCR was binding, or at least dynamically binding. In practice, banks reportedly held a buffer stock of liquid assets above minimum LCR requirements (at roughly 10-20% of the requirement). Adjusting for that would slightly reduce the measured welfare cost. In addition, the level of excess reserves in the measurement period may have been elevated due to the lack of a complete unwind of the Fed's 2008-2014 large scale asset purchases. Focusing on the year least affected by this, 2019, would result in $\lambda = 0.15$. Again, using that value barely changes the measured welfare cost, reducing it just slightly.

[39] We evaluate the quality of the first-order approximation in section 5.5.

[40] For this reason, this risk-adjustment is sometimes referred to as a 'Modigliani-Miller offset.'

Modigliani-Miller theorem fails to hold for them. On this point, empirical analysis by Baker and Wurgler (2015) finds that, while better-capitalized banks have lower risk as expected, lower-risk banks tend to have higher stock returns on a risk-adjusted or even raw basis, so that an increase in capital ratios would result in a (possibly sharply) higher weighted average cost of capital, an outcome that would be qualitatively consistent with the model. Nonetheless, even if the Modigliani-Miller theorem does not hold exactly in reality (as well as in the model), it is still possible that the expected return on equity adjusts to changes in bank leverage, and the empirical approach should take this into account.

Thus, a risk-adjusted measure of the required return on equity is needed from the data. Following Van den Heuvel (2008), we use the average return on subordinated bank debt as a proxy for the risk-adjusted return on equity. The reason for this choice is that (a) subordinated debt counts towards regulatory equity capital, albeit within certain limits, and (b) defaults on this type of debt have historically been rare, so the debt is not very risky, certainly compared to common equity. This proxy avoids the difficulties inherent in measuring the (ex ante) risk premium on common equity,[41] and how that premium adjusts to changes in leverage. Concretely, $(R^E - 1)$ is measured by Interest on Subordinated Notes divided by Subordinated Notes.[42]

The limits on the use of subordinated debt for regulatory purposes as well as its tax treatment imply that this is a conservative measure for the risk-adjusted required return on bank equity. First, subordinated debt can count only towards tier 2 capital, so it only helps to satisfy the risk-based total capital ratio requirement, not the risk-based tier 1, common equity tier 1, or leverage ratio requirements. Second, until the recent adoption of Basel III, the amount of subordinated debt in tier 2 was limited to 50 percent of the bank's tier 1 capital. So if the tier 1 capital ratio was close to binding, subordinated debt could count for at most approximately 25 percent of total capital. Third, relative to common equity, interest on subordinated debt receives favorable corporate income tax treatment. Due to factors, it is possible that for many banks the required return on subordinated debt is lower than the pre-tax, risk-adjusted required return on common equity.

Again, we will use two measurement periods to quantify the welfare cost, one pre- and

[41] For example, the historical average excess return on bank equity would imply a high premium, but does this equal the *ex ante* expected premium? In addition, depending on what interest rate is used to measure the excess return on equity, one would run the risk of contaminating the measured risk premium with a liquidity premium, which one would definitely want to avoid in the present context.

[42] Part of the HSOB's Subordinated Notes does not qualify as regulatory capital. However, cross-checking with the call reports (item RCFD5610) indicates that the difference is minimal after 1992. Also, some subordinated bank debt is callable. Flannery and Sorescu (1996) find that the average call option value for callable bank sub-debt is 0.19%, so the point is minor for the present purpose.

one post-implementation of Basel III. The pre-Basel III period is set to 1993-2006. The start date is motivated by the fact that the first Basel Accord and the FDICIA legislation enacting it were not fully implemented until January 1, 1993, and prior to Basel the use of subordinated debt for regulatory purposes was rather limited.[43] The post-Basel III-implementation period is set at 2016-2019, the same time span as for liquidity regulation.

For 1993-2006, the average nominal interest rates on subordinated debt and deposits are, respectively, 5.87% and 2.67%, so the average cost-revealing spread is 320 basis points. The mean ratio of bank loans to consumption is 0.91 (using total assets minus Treasuries and excess reserves for loans). As explained, the net noninterest cost of servicing deposits is set at 122 basis points and $\lambda = 0$ for this period. Combining these measurements with the analytical result in (19) yields a marginal gross welfare cost of the capital requirement equal to

$$\begin{aligned} \nu_{Cap}|_{\lambda=0} &= (L/c)\left(R^E - R^D - g_D(d, L)\right) \\ &= 0.91 \times (0.0320 - 0.0122) = 0.0180 \end{aligned}$$

Thus, the gross welfare cost of an increase in the capital requirement by 10 percentage points is equivalent to a permanent loss in consumption of about

$$\nu_{Cap} \times 0.1 \times 100\% = 0.18\%$$

This is substantially higher than obtained for a 10 percent liquidity requirement, a point we discuss in more detail below. Further, the cost is similar to the welfare cost of a permanent increase in inflation by a few percentage points, as measured by Lucas (2000). Of course, it should really be compared to the financial stability benefits of such an increase in the capital requirement, analyzed in section 6.

Using the alternative, Hanson et al.-based estimate for the net noninterest cost of deposits ($g_D = 0.81$ percent) yields a modestly larger measurement of the gross marginal welfare cost: $\nu_{Cap} = 0.022$, resulting in a gross welfare cost of 0.22 percent of consumption for an increase in the capital requirement by 10 percentage points.[44]

These measurements for the pre-Basel III period are consistent with Van den Heuvel (2008). Specifically, using a methodology that is also based on subordinated debt as a

[43] The end date excludes the GFC. However, extending this sample period by a few years, or letting it start earlier, has little impact on the results. The contours of Basel III were published in late 2010.

[44] The welfare cost associated with the Corbae and D'Ersamo (2021)-based estimate is always bounded by the costs based on the other two estimates, so for brevity, results presented here will focus on the latter.

proxy for the risk-adjusted required return on equity and a sample period set to 1993-2004, the latter paper estimates the gross welfare cost of a 10 percentage points increase in the capital requirement at 0.10 to 0.22 percent of consumption. This is very similar to the above results, albeit with a wider range that primarily reflects less precise estimates of the noninterest cost of deposits. The similarity is not surprising, as the added generality in the present paper did not alter the sufficient statistic for the welfare cost of the capital requirement conditional on a zero liquidity requirement (see (19)). All that said, Van den Heuvel also employs an alternative, less conservative empirical methodology, which is based on a risk-adjusted return on loans or assets, and those estimates are higher, ranging from 0.7 to 1.5 percent.

Turning to the more recent period, it is worth noting that the U.S. rule implementing Basel III's new capital regime was already finalized in 2013, but its phase-in was an especially gradual one, starting on Jan. 1, 2014, and ending on Jan. 1, 2019. However, in part reflecting anticipation effects and in part banks' desire to pass the stress tests, most of the post-crisis capital buildup, which was substantial, had occurred by the end of 2015.[45] In light of this, and since a sample period starting in 2019 would seem too short, we employ the same Basel III sample period as for liquidity regulation, 2016-2019, even though technically this includes some of the phase in. Reassuringly, shifting the start of the measurement period up or down by one or two years does not have a big impact on the results.

For 2016-2019, the average nominal interest rates on subordinated debt and deposits are, respectively, 3.86% and 0.50%, yielding an average spread of 336 basis points. With liquidity regulation in place in this period, adjustments are needed to use the all-in cost of financing loans with deposits (see (5)) as well as for the noninterest costs of servicing deposits, first set at 1.22%. With $\lambda = 0.17$ now, these adjustments reduce the spread to 206 basis points. The mean loans to consumption ratio is now 1.07. Combining these measurements with the analytical result (18) in proposition 3 yields a marginal gross welfare cost of the capital requirement equal to $\nu_{Cap} = 0.022$. Thus, for the Basel III period, as a first-order approximation, the gross welfare cost of a further increase in the capital requirement by 10 percentage points is equivalent to a permanent loss in consumption of 0.22 percent, a bit more than before Basel III's implementation.

Using the alternative, Hanson et al.-based estimate for the net noninterest cost of

[45] For example, for the largest U.S. banks (global systemically important bank holding companies) the ratio of common equity tier 1 capital to risk-weighted assets increased from 6.3% at the end of the great recession (2009Q2) to 12.0% at year-end 2015, rising only marginally further to 12.2% by the start of 2019. See the Federal Reserve's Financial Stability Report, https://www.federalreserve.gov/publications/financial-stability-report.htm

deposits ($g_D = 0.81\%$) again yields a modestly larger measurement of the gross marginal welfare cost: $\nu_{Cap} = 0.027$, or a gross welfare cost of 0.27 percent of consumption for a 10 percentage points increase in the capital requirement.

5.3 Comparative Assessment

For comparison, table 1 recaps the measurements of the gross welfare costs of both regulations. It presents the permanent consumption loss, in percent, that is to a first-order approximation welfare-equivalent to a 10 percentage point increase in each requirement, for each of the two measurement periods and taking the average across the three estimates of the net noninterest costs of servicing deposits. The numbers below, in parentheses, represent the full range depending on these cost estimates.

The main comparative takeaway from table 1 is that the welfare cost of an increase in the capital requirement is roughly ten times as large as the cost of a similarly sized increase in the liquidity requirement. The divergence by an order of magnitude appears to hold regardless of the measurement period.

This key result reflects an insight obtained from the model: capital requirements reduce the supply of safe, liquid assets available to the public by much more than liquidity requirements do. Even with the general equilibrium feedbacks that change the size of the banking sector,[46] capital requirements effectively reduce the supply of bank deposits, replacing them to an important degree with bank equity, an instrument that does not provide liquidity services. In contrast, liquidity requirements effectively transform some government bonds held by the public into bank deposits (which fund the banks' holdings of government bonds). These are both liquid instruments that command a convenience yield, so the *net* reduction in liquidity services available to the nonbank public is much smaller. In the data, this is manifested by a smaller spread between Treasuries and bank deposits than between equity and deposits, and that is why the welfare cost estimates differ as much as they do. Of course, these costs should ultimately be compared to the benefits of the requirements, a topic we take up in section 6.

[46] Indeed, the (potentially large) changes in the size of the banking sector, the capital stock, and consumption – taking into account changes in both the transition and the steady state – have a *combined* effect on welfare that is second-order. Mathematically, this is a manifestation of the envelope theorem.

Table 1. Gross Welfare Costs of the Liquidity and Capital Requirements

	Measurement period	
Welfare cost of:	Pre-Basel III	Basel III
10% LIQUIDITY requirement	0.022	0.019
	(0.011-0.038)	(0.001-0.046)
10% CAPITAL requirement	0.202	0.250
	(0.180-0.217)	(0.219-0.272)

Note: Entries are first-order approximations of the permanent consumption loss, expressed in percent, that is welfare-equivalent to a 10 percentage point increase in each requirement, holding fixed the incidence of financial crises, and taking the average across estimates of the net noninterest costs of servicing deposits (0.81%, 0.90%, and 1.22%). Numbers in parentheses indicate the full range of measured welfare costs, with the upper end associated with high (low) net noninterest costs for the liquidity (capital) requirement. 'Pre-Basel III' means 2001-2006 for liquidity and 1993-2006 for capital. 'Basel III' means 2016-2019.

A secondary finding that is apparent from the table is that the gross marginal welfare cost of the capital requirement is higher now than in the pre-Basel III period, by 24 percent based on the period-specific averages. Since Basel III raised capital requirements, this finding is consistent with increasing marginal costs, i.e. convex welfare costs. In contrast, the marginal welfare cost of liquidity regulation is about the same across the periods. From an accounting perspective, the rise in the measured cost of the capital requirement reflects increases in the loans to consumption ratio and in the cost-revealing equity-deposit spread, which outweighed the adjustments for a higher liquidity requirement (see (18)).[47]

5.4 Narrow Banking

A narrow bank is a depository institution that holds only safe, liquid assets. Proponents of narrow banking argue that such firms would be (virtually) immune to bank runs and failures, thus eliminating the economic harm caused by failures of deposit-taking firms. Loans to firms and households would instead be made by non-deposit-taking financial firms,

[47] As shown in Begenau (2020) and Van den Heuvel (2006), the general equilibrium effect of an increase in the capital requirement on bank lending can be positive. While perhaps surprising, this outcome occurs when the interest-elasticity of the demand for deposits is low. In that case, an increase in the capital requirement leads to an increase in the deposit spread ($R^E - R^D$) that is sufficiently large to reduce banks' lending rate (R^L), despite the costlier funding mix.

i.e. nonbank financial intermediaries, or would be replaced by market-based finance, such as bonds or commercial paper.

In our model, narrow banking is already permitted: a bank can become a narrow bank by maintaining a balance sheet with only government bonds and deposits. Moreover, other banks could opt to operate like nonbanks by making only equity-financed loans, and firms in the model can also borrow from actual NBFIs. For banks, neither business model would violate regulatory constraints.[48] However, the data suggest that these strategies are not always profitable. The narrow bank is not profitable whenever the liquidity requirement binds (that is, whenever $R^D + g_D > R^B$) or, equivalently, whenever there is a positive welfare cost of liquidity regulation, which we have found to be the case for at least some periods.[49] The all-equity 'bank' is not profitable whenever the capital requirement binds (that is, whenever $R^L - g_L < R^E$) or, equivalently, whenever there is a positive welfare cost of the capital requirement - again, we have found this to be true in the data.

What then would be the gross welfare cost of *requiring* deposit-taking institutions to be narrow banks? In the model, this can be achieved by imposing 100% liquidity and capital requirements.[50] We can gauge the cost of such a policy in the same way as we have measured the costs of smaller policy changes above. It must be stated at the outset, though, that doing so may come with significant approximation error: our measurements rely on first-order approximations, which may not perform well for such a big policy change.

If we nonetheless proceed with that caveat in mind, relative to current regulations, the gross welfare cost of requiring deposit-taking institutions to be narrow banks is (very) roughly equal to $\nu_{Liq} \times (1 - 0.17) + \nu_{Cap} \times (1 - 0.12) = 2.4\%$ of consumption.[51] Not surprisingly, a move to narrow banking would be considerably costlier than the smaller policy changes contemplated in table 1. Intuitively, narrow banking is costly because it ignores the reality that, despite the demandable nature of many deposits, most depositors

[48] This reflects the absence of a leverage ratio restriction from the model. A leverage ratio rule would require the narrow bank to maintain some equity against its government bonds.

[49] Hanson, Schleifer, Stein, and Vishny (2015) reach a similar conclusion by comparing the total cost of deposits to the return on T-bills (or, in our notation, $R^D + g_D$ to R^B).

[50] Technically, such a policy ($\lambda = 1$ and $\gamma = 1$) would not impose the separation of narrow banks and finance companies. This is without loss of generality if $g = 0$ or if g is separable in its arguments. However, if g is not zero nor separable, then its assumed convexity and linear homogeneity imply that there are cost savings from combining the narrow bank and financing company in a single firm ($g(D, L) \leq g(D, 0) + g(0, L)$). In that case, the calculations that follow will miss some of the costs associated with a move to narrow banking.

[51] The baseline level of the capital requirement is set at 0.12, which is the current ratio of common equity tier 1 capital to risk-weighted assets of the largest U.S. bank holding companies, and the baseline liquidity requirement is set at its current level, 0.17 (see section 5.1 for details). The calculation is based on the average welfare costs for the Basel III period, as reported in table 1 for 10 p.p increases in each requirement. Using the averages from the pre-Basel III measurement period results in an estimated welfare cost of 2.0%.

do not withdraw their funds all the time, allowing banks to use a portion of their deposits to fund more illiquid loans, creating more net liquidity and reducing the cost of loans.

As noted, this number should not be viewed as a precise estimate. In the next subsection, we evaluate the quality of the first-order approximations, including for narrow banking, finding an exact effect for a calibrated economy that is modestly higher.

5.5 Accuracy of the First-Order Approximations

As mentioned, for discrete changes in the requirements, the quantitative results obtained so far are first-order approximations based on the marginal welfare costs. To evaluate the quality of those approximations, the model economy is calibrated and solved numerically, using the global nonlinear method of Mendoza and Villalvazo (2020), to obtain exact numbers for the calibrated economy. Of course, this involves specifying all functional forms and parameter values, as detailed in Appendix D. Briefly, standard choices are made where possible, while the remaining parameters are picked to match the various measurements of the spreads and ratios in the expressions for the *marginal* welfare costs, as presented in this section. By construction, therefore, any differences between the exact and the first-order approximate welfare costs are due to error in that approximation.

The results are reassuring. The numbers in table 1 are within 1 to 3 basis points of the corresponding exact number in the calibrated model economy. As expected, for smaller changes in each requirement, such as 1 percentage point up or down, the exact number and the first-order approximation are virtually identical. For the very large change of a narrow-banking requirement, the exact welfare cost is 3.0 percent of consumption, compared to the 2.4 percent found above. A number of robustness checks do not significantly alter these conclusions (see the appendix for details).

6 Bank Risk and the Benefits of Regulation

This section turns to the welfare benefits of liquidity and capital requirements. So far, the analysis has focused on their welfare costs, based on a version of the model with minimal assumptions. To examine benefits, we first present an expanded version of the model, in which banks are exposed to risks and fail with positive probability. Because bank failures entail externalities, a role for regulation arises endogenously, allowing for an analysis of its benefits as well as its costs. With respect to the latter, it will be shown that the formulas for the welfare costs of the liquidity and capital requirements presented in section 4 remain valid as gross costs in the expanded model. Likewise, their measurements in section 5 remain unaffected by the consideration of bank risk and regulatory benefits.

6.1 The Expanded Model

The expanded version of the model adds bank liquidity and credit risk. The presence of deposit insurance means that banks do not fully internalize all the costs of downside risks, resulting in distorted decisions and a role for bank regulation. The following features are added to the model presented in section 2. Unless stated otherwise, all the assumptions stated in that section continue to hold.

6.1.1 Firms

To introduce credit risk, output from production is now assumed to be uncertain and given by

$$F(K_t, H_t) + \varepsilon_t K_t$$

where ε_t is an *i.i.d.* sectoral[52] shock to the productivity of capital. Each firm operates in a specific sector, which can also be interpreted as a geographic area. There is a mass-one continuum of sectors and a mass-one continuum of firms in each sector. Without loss of generality, it is assumed that $\mathbb{E}[\varepsilon] = 0$. Further, ε has a cumulative distribution function F_ε, with bounded support $[\underline{\varepsilon}, \bar{\varepsilon}]$. The shock ε_t is realized after choices for K_t and H_t have been made, and ε_t is observable once realized and is contractible. As before, capital is financed with a combination of bank and nonbank loans: $K_t = L_t + L_t^{NB}$.

As shown in Appendix C.1, maximizing expected profits yields the same standard first-order conditions for labor and capital as before – see (8) and (9) – provided we adopt the convention that with uncertainty, R_t^L represents the *expected* loan rate: $R_t^L \equiv \mathbb{E}[R_t^L(\varepsilon_t)]$, where without loss of generality, $R_t^L(\varepsilon_t)$ is the firm's stochastic loan repayment rate. The appendix derives the full schedule for $R_t^L(\varepsilon_t)$, which is pinned down by free entry and limited liability of the owners of the firm. It also shows how this schedule can be implemented through a state-contingent loan contract or equivalently through a standard loan contract with a fixed contractual interest rate and firm default.

[52] The reason for this assumption will become clear in the subsection on banks. Adding a firm-specific shock to the productivity of capital would introduce more complexity without changing the results.

6.1.2 Banks

This subsection starts with a description of the additional assumptions regarding banks in the expanded model. These add features – deposit insurance, credit risk, and liquidity risk – that shape the moral hazard problem of excessive risk taking, leading to bank failures.

Deposit insurance A government-run deposit insurance fund ensures that no depositor suffers a loss in the event of a bank failure. That is, all deposits are fully insured. The rationale for the deposit insurance is left unmodeled. However, it has been argued that deposit insurance improves the ability of banks to create liquidity.[53] Deposit insurance creates a moral hazard problem: the bank has an incentive to engage in excessive risk taking, which provides a rationale for capital and liquidity regulation. The expanded model features two sources of bank risk: credit risk and liquidity risk.

Credit risk Consistent with the technology of nonfinancial firms, loans yield a gross rate return of $R_t^L + \varepsilon_t$ at the end of the period t, where the shock ε_t is a sectoral shock that is *i.i.d.* across sectors and over time - the same shock that affects firms' productivity of capital. As for firms, R_t^L is defined as the *expected* loan return net of charge-offs, so ε_t has zero mean. Other than bounded support, no assumptions are imposed on the distribution of ε. As noted, these assumptions are consistent with standard loan contracts and firm default. To the bank, ε_t is credit risk that it is not able to diversify away, which could reflect sectoral or geographic specialization. Formally, each bank lends to a specific sector, and there is a continuum of banks in each sector, with mass normalized to one.

Liquidity risk Deposits come with liquidity risk for the bank. This feature is introduced not just for realism, but also to provide a rationale for liquidity regulation. It is assumed that a fraction w_t of depositors decide to withdraw early, where w_t is a bank-specific exogenous shock that is *i.i.d.* across banks and over time, and ε_t and w_t are mutually independent. It is natural to assume that w does not exceed one, so denoting its distribution function by F_w, $F_w(1) = 1$. In addition, it is convenient to assume that $F_w(0) > 0$ (positive probability of no liquidity stress) and that F_w is continuously differentiable on $(0, 1]$.

The bank can cover these early withdrawals by drawing down its stock of liquid securities, i.e. its holdings of government debt. In contrast, loans are fully illiquid and no secondary market exists for loans. As a consequence, if the bank does not have sufficient government bonds to cover the intra-period withdrawals, the bank defaults and goes into

[53] Diamond and Dybvig (1983) provide a model of liquidity provision by banks, in which socially undesirable, panic-based bank runs can occur, and in which deposit insurance can prevent these runs.

resolution. Shareholders get zero in this case, while depositors are made whole by the deposit insurance fund. The resolution of failed banks is discussed in more detail below. The assumption of complete illiquidity of loans is admittedly an extreme one. The key idea, however, is that loans are less liquid, especially in times of stress, and that this can make it desirable for banks to hold more liquid securities in anticipation of deposit outflows. The question then becomes whether the private incentives to hold liquid assets are as strong as the social benefits.

It is assumed that early withdrawers use their funds to make payments to other households, who then deposit the funds into the banking system. To economize on notation and avoid having to keep track of intra-period balance sheet changes, we adopt the simplifying assumption that those banks that experienced the liquidity outflows are also shortly thereafter recipients of liquidity inflows of the same magnitude (regardless of whether they survived the acute liquidity stress or are in FDIC resolution). Although this is clearly not the most realistic assumption, it simplifies the analysis and more realistic assumptions would lengthen the exposition without yielding additional insights.

Note that the expanded model nests the baseline model through setting $\varepsilon = 0$ and $w = 0$ with probability 1.

The bank's decision problem The bank's objective is to maximize shareholder value, net of the initial equity investment:[54]

$$\pi^B = \max_{L,B,D,E} \mathbb{E}\left[1_{\{B \geq wD\}}\{(R^L + \varepsilon)L + R^B B - R^D D - g(D,L)\}^+\right]/R^E - E \tag{22}$$

The notation $\{x\}^+$ stands for $\max(x, 0)$ and $1_{\{B \geq wD\}}$ is an indicator variable taking the value 1 if $B \geq wD$ and zero otherwise, reflecting the fact that the bank will fail due to liquidity stress if $B < wD$. The constraints are, as before, the balance sheet, the capital requirement, and the liquidity requirement; see (4).

The term $(R^L + \varepsilon)L + R^B B - R^D D - g(D, L)$ is the net cash flow at the end of the period, provided there was no failure due to liquidity stress. If this net cash-flow is positive, shareholders are paid this full amount in dividends. If it is negative, the bank fails and the deposit insurance fund must cover the shortfall to indemnify depositors, as limited liability of shareholders rules out negative dividends. Shareholders receive zero in this event or if the bank has already failed due to liquidity stress, so dividends equal the expression inside

[54] As before, each bank is potentially long-lived, but due to the lack of adjustment costs and agency frictions with the other optimizing agents, its decision problem is effectively static. In what follows, time subscripts will be used only where necessary to avoid confusion.

the square brackets. As before, shareholders discount the expected value of end-of-period dividends by R^E, the market return on equity. While an individual's bank's dividends are risky, this risk is perfectly diversifiable for shareholders, so they do not price it.[55]

It is straightforward to solve this problem (see Appendix C.2). To summarize the results, it is convenient to introduce the notation

$$\bar{w} \equiv B/D \tag{23}$$

$\bar{w}$ is the failure threshold for w. That is, a realization of w that exceeds $\bar{w}$ results in failure from liquidity stress, as $B < wD$ in that case. The liquidity regulation requires that $\bar{w} \geq \lambda$, and the desire to self-insure against the risk of liquidity-driven failure may or may not result in that inequality being strict. Adapting the notation introduced in section 2,

$$\tilde{R}^D(\bar{w}) \equiv R^D + \frac{\bar{w}}{1-\bar{w}}(R^D - R^B) \tag{24}$$

$\tilde{R}^D(\bar{w})$ is the all-in cost of financing loans with deposits, taking into account that the bank puts a fraction $\bar{w}$ of the deposits raised in bonds (but setting aside noninterest costs g).

In addition, to characterize the failure threshold from credit risk, define r_L as net cash flow conditional on $\varepsilon = 0$, no failure due to liquidity stress, and normalized by loans:

$$r_L \equiv R^L + R^B(B/L) - R^D(D/L) - g(D/L, 1) \tag{25}$$

Using that notation, the bank fails due to credit risk if $\varepsilon < -r_L$. The banks' shareholders will care about loan returns *outside* bankruptcy. Accordingly, it will be convenient to define a distorted expected loan return as follows:

$$R^{L,d} \equiv R^L + \mathbb{E}\left[\varepsilon | \varepsilon > -r_L\right] \geq R^L \tag{26}$$

where the inequality follows from $\mathbb{E}\left[\varepsilon\right] = 0$. With that, the following proposition summarizes the solution:

[55] Hence, the treatment of R^E as nonstochastic in the household problem is also still correct, since, even though banks are risky, households would not leave any such risk undiversified.

Proposition 4 (solution to the bank's problem with credit and liquidity risk) *The bank fails with probability*

$$p_F = 1 - F_w(\bar{w})(1 - F_\varepsilon(-r_L)) \in [0, 1) \tag{27}$$

Conditional on no failure, the expected return on equity is

$$\hat{R}^E \equiv R^E/(1 - p_F) \geq R^E \tag{28}$$

The solution satisfies a distorted zero-profit condition:

$$R^{L,d} - g_L(D, L) = \gamma \hat{R}^E + (1 - \gamma)\left\{\tilde{R}^D(\bar{w}) + \frac{1}{1 - \bar{w}} g_D(D, L)\right\} \tag{29}$$

resulting in $\pi^B = 0$*. If the liquidity requirement binds,* $\bar{w} = \lambda$*. A finite solution requires*

$$R^B + z \leq R^D + g_D(D, L) + \bar{w}z \leq R^{L,d} - g_L(D, L) \leq \hat{R}^E \tag{30}$$

where $z \equiv \frac{F'_w(\bar{w})}{F_w(\bar{w})} \frac{\hat{R}^E E}{D} \geq 0$ *and* $\lambda \leq \bar{w} \leq 1$*. The liquidity requirement binds (*$\bar{w} = \lambda$*) if and only if the first inequality in (30) is strict. If the liquidity requirement is slack, then* $\bar{w}$ *is determined by* $(1 - \bar{w})z = R^D + g_D - R^B$*. The capital requirement binds if and only if the last inequality is in (30) strict, or equivalently, if and only if* $\tilde{R}^D(\bar{w}) + \frac{1}{1-\bar{w}} g_D < \hat{R}^E$*.*

Proof: See Appendix C.2.

The key difference with the baseline model (proposition 1) is the presence of two distortions, reflecting the interplay of risk, limited liability of shareholders, and deposit insurance. First, since the returns to lending and bond holdings as well as the costs of deposits are recouped or incurred by shareholders only if the bank does not fail, these returns and costs are compared to the required return to shareholders conditional on no failure, which is $\hat{R}^E$. Moreover, for the same reasons, the relevant expected lending return is $R^{L,d}$, the expected return conditional on no failure. This level exceeds R^L, the unconditional expected return. Thus, in the presence of risk, deposit insurance boosts incentives to invest more in risky assets (loans) and to increase leverage (as $\hat{R}^E \geq R^E$). We refer to the former incentive as the **risk-shifting distortion** and to the latter as the **leverage distortion**.

These distortions have opposing effects on the lending rate. To illustrate this, assume for a moment that there are no noninterest costs ($g = 0$) and that the liquidity requirement binds (so $\bar{w} = \lambda$). Then, using (26), the zero-profit condition (29) simplifies to

$$R^L = \gamma \hat{R}^E + (1-\gamma)\tilde{R}^D(\lambda) - \mathbb{E}\left[\varepsilon | \varepsilon > -r_L\right]$$

This right-hand side exceeds the baseline result in (7) by $\gamma(\hat{R}^E - R^E) - \mathbb{E}\left[\varepsilon|\varepsilon > -r_L\right]$. If the probability of failure due to credit risk is strictly positive, then $\mathbb{E}\left[\varepsilon|\varepsilon > -r_L\right] > 0$, indicating an operative risk-shifting motive, which leads to a lower lending rate and thus more lending in equilibrium. At a low capital requirement γ, the risk-shifting distortion is likely to dominate the higher private cost of equity finance associated with the leverage distortion, seen in $\hat{R}^E - R^E$ (which is positive whenever the failure probability is positive; see (28)). Raising the capital requirement can counter this risk-shifting incentive, however.

The capital requirement still binds if equity is more expensive than deposits, but now taking into account the leverage distortion as well as the noninterest costs of deposits and the need to respect the liquidity requirement or maintain $\bar{w}$, whichever case applies.

The liquidity requirement now binds if

$$R^D + g_D(D, L) - R^B > (1-\bar{w})\frac{F_w'(\bar{w})}{F_w(\bar{w})}\frac{\hat{R}^E E}{D} \tag{31}$$

The right-hand side is the benefit to shareholders of holding marginally more bonds, financed with deposits, in terms of reducing probability of failure from liquidity stress (see the appendix for details). The left-hand side is the cost of such a balance sheet expansion, and the liquidity requirement binds if the costs exceed the benefits. In the baseline version of the model, the liquidity requirement was binding simply when the costs exceeded zero (proposition 1). As the benefits here (right-hand side of (31)) are positive, the liquidity requirement is less likely to bind with liquidity risk, reflecting the self-insurance incentive.

Proposition 4 can be used to characterize the effect of the capital and liquidity requirements on the probability of failure. Specifically, the following result derives from (27) and zero profits. As it holds fixed the required expected return on equity, it can be viewed as a partial equilibrium result. That said, even in general equilibrium, this assumption holds across steady states (as $R^E = \beta^{-1}$ in any steady state, a level that does not depend on the regulations).

Proposition 5 *Assume the capital and liquidity requirement both bind and hold fixed the required expected return on equity, R^E. Then*

- *An increase in the capital requirement γ reduces the probability of failure from credit risk, $F_\varepsilon(-r_L)$; has no impact on the probability of failure from liquidity risk, $1-F_w(\lambda)$; and thus reduces the overall probability of failure, p_F.*

- *An increase in the liquidity requirement λ increases the probability of failure from credit risk, $F_\varepsilon(-r_L)$; reduces the probability of failure from liquidity risk, $1-F_w(\lambda)$; and thus has an ambiguous effect on the overall probability of failure, p_F.*

Proof: See Appendix C.2.

Both capital and liquidity regulations are helpful in mitigating the moral hazard from deposit insurance and thereby preventing banking failures and financial crises. The capital requirement is the only tool that reduces failures from credit risk, while the liquidity requirement is the only tool that reduces failures from liquidity stress in this setting. Thus, the model indicates a simple division of labor: let the capital requirement deal with credit risk and use the liquidity requirement for liquidity risk.

It is worth noting that the analysis suggests an adverse side effect of liquidity regulation: by adversely impacting the ability of banks to operate profitably, a higher binding liquidity requirement actually raises the probability of failure from credit risk, leading to a theoretically ambiguous overall impact on the overall probability of bank failure. The finding that the liquidity requirement exacerbates credit risk may seem surprising, since the regulation forces the bank to hold more safe assets (government bonds), which would seem to reduce its overall credit risk. However, bank size is not fixed, and a higher liquidity requirement does not force the bank to reduce its loan portfolio – it can simply raise more deposits and invest the proceeds in bonds, leaving the scope for risk taking through lending unchanged. All that said, the key point is that both requirements are needed if the goal is to reduce the risks of both solvency- and liquidity-driven bank failures.

6.1.3 Other Agents

The decision problems of the other agents remain largely the same. For households, to streamline the analysis, we assume strict monotonicity of liquidity preferences (i.e. $u_d(c,d,b) > 0$ and $u_b(c,d,b) > 0$). Despite the introduction of risk, the return on equity R^E remains deterministic for households as bank and sectoral risk is fully diversified.

Similarly, NBFIs can fully diversify sectoral credit risk, so its decisions are unaffected.[56] Thus, the optimality conditions in the baseline model for households and NBFIs, shown in (1), (2), (3) and (10), remain the same in the expanded model.

Reflecting the incidence of bank failures, the model allows for a resource cost arising from the resolution of failed banks by the government. $\psi_{Liq} \geq 0$ and $\psi_{Sol} \geq 0$ denote the deadweight resolution costs per unit of loans in banks that fail due to liquidity stress or insolvency, respectively. Total deadweight resolution costs are thus

$$\Psi_t \equiv (1 - F_w(\bar{w}_t))\psi_{Liq} L_t + F_w(\bar{w}_t) F_\varepsilon(-r_{L,t})\psi_{Sol} L_t \tag{32}$$

and lump-sum taxes are now

$$T_t = (R_t^B - 1)\bar{B} + \Psi_t - (1 - F_w(\bar{w}_t)) r_{L,t} L_t - F_w(\bar{w}_t) \int_{\underline{\varepsilon}}^{-r_{L,t}} (r_{L,t} + \varepsilon)) L_t dF_\varepsilon(\varepsilon) \tag{33}$$

The right-most terms are the gains/losses from the operation of failed banks in resolution by the deposit insurance fund.

6.1.4 General Equilibrium

In contrast to the baseline model, the equilibrium now features positive rates of bank failures from credit and liquidity risk (propositions 4 and 5). These bank failures not only lead to resolution costs but also distort banks' decision *ex ante*. As explained, due to deposit insurance, banks have an incentive to take more risk by lending more and to lever up as much as possible. Consequently, equilibrium investment is also distorted if banks are the marginal lenders (as happens in a pure bank finance equilibrium). The next subsection will explore the normative implications of these distortions in more detail. Otherwise, the equilibrium largely resembles the baseline model (see Appendix C.3 for technical details). While the capital requirement continues to bind, the liquidity requirement is now somewhat less likely to bind, as banks have an incentive to self-insure against liquidity stress by holding government bonds (proposition 4). Both requirements can still impact investment, and the equilibrium may be one of pure bank finance or mixed finance with NBFI lending. Finally, while very stringent regulation can still drive financial activity to nonbanks, it now has the benefit that it tends to reduce bank failures and associated resolution costs (proposition 5).

[56] While convenient, the assumption that NBFIs can lend across sectors is not essential, as sectoral risk is not systematic and would thus not be priced by the NBFI's shareholders.

6.2 The Welfare Benefits and Costs with Bank Risks

To characterize the welfare benefits and costs of the regulatory requirements, we will again use a social planner's problem that is designed to replicate the competitive equilibrium, rather than solve for the first best. Because of the distortions present in that equilibrium, replication will require the use of several wedges, denoted by τ. Specifically, the constrained social planner's problem is defined as follows:

$$V_0(\theta) = \max_{\{c_t,d_t,b_t,B_t,L_t,E_t,K_{t+1}\}_{t=0}^{\infty}} \sum\nolimits_{t=0}^{\infty} \beta^t u(c_t, d_t, b_t) \tag{34}$$

$$\text{s.t. } K_{t+1} = F(K_t, 1) + (1-\delta)K_t - c_t - g(d_t, L_t) - \varphi(K_t - L_t) + \tau_t^L(\theta)L_t - \tau_t^E(\theta)E_t + Q_t(\theta),$$

$$B_t + b_t = \bar{B},\ E_t + d_t = L_t + B_t,\ B_t = (\lambda + \tau_t^{\bar{w}}(\theta))d_t,\ E_t \geq \gamma L_t,\ K_t \geq L_t$$

where $\theta = (\lambda, \gamma, K_0)$ and the social planner takes the following variables as given:

$$\begin{aligned} \tau_t^E(\theta) &\equiv \hat{R}_t^{E,ce} - R_t^{E,ce} \\ \tau_t^L(\theta) &\equiv \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}^{ce}\right] \\ \tau_t^{\bar{w}}(\theta) &\equiv \bar{w}_t^{ce} - \lambda \\ Q_t(\theta) &\equiv \tau_t^E(\theta)E_t^{ce} - \tau_t^L(\theta)L_t^{ce} - \Psi_t^{ce}(\theta) \end{aligned} \tag{35}$$

Here, the superscript *ce* is used to denote competitive equilibrium values, all of which the planner takes as given, and all of which may depend on θ. The terms $\tau_t^L L_t$ and $-\tau_t^E E_t$ in the first constraint insert distortions into the social planner's problem, which are necessary to replicate the competitive equilibrium (CE). The wedge τ_t^L captures the risk-shifting distortion that results in overlending, while the wedge τ_t^E captures the leverage distortion. The term Q_t 'returns' $\tau_t^E E_t - \tau_t^L L_t$ at CE values, which the planner takes as given, to ensure that the social resource constraint holds.[57] Similarly, Q_t contains Ψ_t^{ce}, the CE's resolution costs, defined in (32). Just like the optimizing agents in the CE, the planner does not take into account the effect of her choices on resolution costs.

The social planner has a time-varying liquidity requirement, which equals λ whenever the liquidity requirement binds in CE, and equals $\bar{w}_t^{ce}(\theta) > \lambda$ otherwise. This simplifies the social planner problem by abstracting from explicit modelling of bank-specific liquidity risk. It may seem odd to view $\tau_t^{\bar{w}} \equiv \bar{w}_t^{ce} - \lambda$ as a wedge, but it is logically coherent. Socially, the only costs of liquidity-driven bank failures are their resolution costs. The private incentives

[57] The planner's constraints correspond to the social resource constraint, bond market clearing, bank balance sheets, the liquidity and capital requirements, and nonnegativity of NBFI lending, in that order.

to self-insure against the risk of such failures by choosing $\bar{w}_t^{ce} > \lambda$ are driven by the transfer of shareholder value to the deposit insurance fund. That transfer may or may not be sufficient to cover the resolution costs, but it is a transfer, not a social loss.

Appendix C.4 shows that the allocation of this planner is identical to the decentralized equilibrium. Therefore, welfare in that equilibrium is equal to $V_0(\theta)$, making it straightforward to derive expressions for the marginal welfare effects of the two regulations by differentiating $V_0(\theta)$ with respect to λ and γ, using the envelope theorem. These derivatives have several terms. Terms reflecting reduced liquidity creation are classified as welfare *costs*, while terms related to reductions in resolution costs and distortions are classified as welfare *benefits*. The results are presented in the next two propositions.

Proposition 6 (welfare costs and benefits of the liquidity requirement) *With risky banks, the sufficient statistic for the marginal welfare cost of a binding liquidity requirement remains the same as in the baseline model (proposition 2). The marginal welfare benefit of the liquidity requirement, expressed as the welfare-equivalent permanent relative gain in consumption, is*

$$\sum_{t=0}^{\infty} \frac{\varpi_t}{c_t} \left\{ \tau_t^E \frac{\partial E_t}{\partial \lambda} - \tau_t^L \frac{\partial L_t}{\partial \lambda} - \frac{\partial \Psi_t}{\partial \lambda} \right\} \tag{36}$$

where $\varpi_t \equiv \frac{\beta^t u_c(c_t,d_t,b_t)c_t}{\sum_{s=0}^{\infty} \beta^s u_c(c_s,d_s,b_s)c_s}$ *is a series of positive weights that sum to one.*

Proof: See Appendix C.4.

The most important part of this proposition is its first sentence: The sufficient statistic for the marginal welfare cost of a binding liquidity requirement remains the same as in the baseline model (stated in proposition 2), showing the robustness of this central result, which is driven by fundamental economic forces. The robustness means that the measurements of the welfare cost of the liquidity requirement provided in the previous section also remain valid for the economy with bank risks. Moreover, as before, we have imposed only minimal assumptions on preferences and other functional forms (for noninterest costs, $g(D, L)$, production, $F(K, H)$, and now also for the shock distributions, $F_w(w)$ and $F_\varepsilon(\varepsilon)$). In addition, the formula applies whether bank loans are special or not, whether the equilibrium is entails pure bank finance or mixed finance, and it takes into account gains and losses associated with the transition to a new steady state. The intuition also remains the same: A binding liquidity requirement essentially removes Treasuries from nonbank investors and puts them in banks, but banks can finance these new assets with deposits which, like Treasuries, also provide liquidity services. The spread between these two instruments reveals the magnitude of the net social cost.

The marginal welfare benefits are characterized in (36). Each component reflects an externality. First, the banks' leverage distortion is reduced when banks use more equity. (Note that τ_t^E and τ_t^L are both positive.) Second, banks' overlending distortion is reduced when banks lend less as a result of a higher liquidity requirement. The third term is the reduction in resolution costs from an increase in the liquidity requirement. Because the benefits reflect these externalities, they do not appear amenable to a sufficient statistics approach based on revealed preference arguments, in contrast to the welfare costs. Instead, quantifying the derivatives in the formula would require additional assumptions; for example, on the magnitude of the resolution costs, ψ_{Liq} and ψ_{Sol}, and the shapes of the derived utility function u and the distribution functions for the shocks, F_ε and F_w.[58] Despite these challenges, we take such an approach in the next subsection.

The next proposition characterizes the costs and benefits of the capital requirement in the expanded model.

Proposition 7 (welfare costs and benefits of the capital requirement) *With risky banks, the sufficient statistic for the marginal welfare cost of the capital requirement remains the same as in the baseline model (proposition 3). The marginal welfare benefit of a binding capital requirement, expressed as the welfare-equivalent permanent relative gain in consumption, is*

$$\sum\nolimits_{t=0}^{\infty} \frac{\varpi_t}{c_t} \left\{ (\gamma \tau_t^E - \tau_t^L) \frac{\partial L_t}{\partial \gamma} - \frac{\partial \Psi_t}{\partial \gamma} + z_t \left(\frac{\tau_t^{\bar{w}} L_t}{1-\lambda} - d_t \frac{\partial \tau_t^{\bar{w}}}{\partial \gamma} \right) \right\} \tag{37}$$

where $\varpi_t \equiv \frac{\beta^t u_c(c_t,d_t,b_t)c_t}{\sum_{s=0}^{\infty} \beta^s u_c(c_s,d_s,b_s)c_s}$ *is a series of positive weights that sum to one.*

Proof: See Appendix C.4.

As for the liquidity requirement, the formula for the marginal welfare cost of the capital requirement remains the same as in the baseline model (stated in proposition 3). Again, this demonstrates the robustness of this key result, and it also means that the measurements of the welfare cost of the capital requirement provided in the previous section are equally valid for the economy with bank risks. Further, the sufficient statistic applies with the same generality as described for the liquidity requirement, and the basic intuition remains as in the baseline: the capital requirement entails a gross welfare cost because it constrains the ability of banks to issue deposit-type liabilities, which are valued for their liquidity.

[58] In fact, as illustrated by proposition 5, it is generally not possible to even sign all the relevant effects without additional assumptions. For the same reason, a classification scheme where say, positive welfare effects would be classified as benefits and negative one as costs is not implementable at this level of generality.

The marginal welfare benefits of the capital requirement are characterized in (37). As for the liquidity requirement, each component reflects an externality. The leverage distortion is reduced when banks use more equity as a result of a higher capital requirement, while the overlending distortion is reduced when banks lend less. The reduction in resolution costs from an increase in the capital requirement is also included in the benefits. The right-most terms reflect the liquidity wedge, $\tau_t^{\bar{w}}$. When the liquidity requirement binds, these terms drop out, since $\tau_t^{\bar{w}} = 0$ and $\partial \tau_t^{\bar{w}} / \partial \gamma = 0$ in that case. Hence, under a binding liquidity requirement, the marginal welfare benefits of the capital requirement simplify to

$$\sum\nolimits_{t=0}^{\infty} \frac{\varpi_t}{c_t} \left\{ (\gamma \tau_t^E - \tau_t^L) \frac{\partial L_t}{\partial \gamma} - \frac{\partial \Psi_t}{\partial \gamma} \right\} \tag{38}$$

Again, because the benefits reflect these externalities, they do not appear amenable to a sufficient statistics approach based on revealed preference arguments. The next subsection takes a different route to quantification.

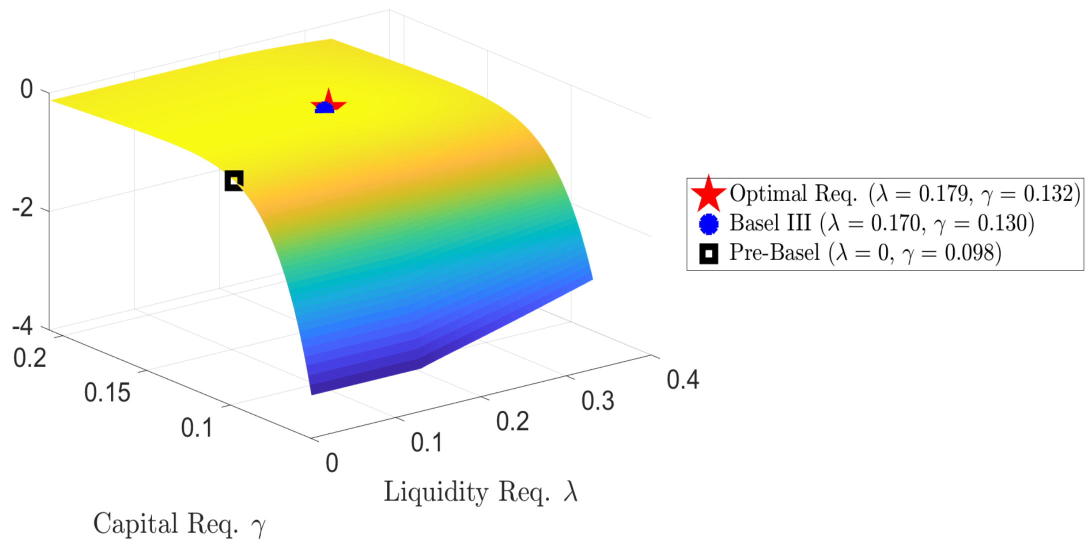
6.2.1 Quantifying the Welfare Benefits and Optimal Policy

To quantify the welfare benefits and the optimal levels of the capital and liquidity requirements, we fully calibrate and numerically solve the expanded model. The calibration and solution are detailed in Appendix E. To summarize briefly, the calibration is similar to the one used in section 5.5 to evaluate the baseline model, with standard parameter values where possible and other parameters chosen to match the observed spreads and ratios in the expressions for the marginal welfare costs, which remain valid in the expanded model, as shown. This strategy ensures that the quantification of the benefits is consistent with the measurements of the costs in section 5. For the distributions of liquidity and credit risk, we match relevant moments from call reports and the rate of bank failures.[59] Resolution costs are calibrated to match losses to the FDIC's Deposit Insurance Fund.

Based on this, we find that the marginal welfare benefits of the capital requirement are far larger than for the liquidity requirement, echoing the results on the costs. There are two reasons for this finding. First, the calibration implies that most bank failures – the occurrence and prospect of which create adverse externalities and distortions– stem from credit rather than liquidity risk, a result that is consistent with the empirical findings of Correia, Luck, and Verner (2026). Further, as shown in proposition 5, only the capital

[59] Our main calibration matches to the bank failure rate calculated by Corbae and D'Erasmo (2021) for the decades preceding Basel III (1.07%). Our calibration is consistent with that timing. In particular, it accounts for the fact that requirements were lower before Basel III.

Figure 3: Welfare



Level of welfare expressed in percent consumption equivalents relative to Basel III

requirement reduces failures driven by credit risk, leading to substantial welfare benefits that are absent for liquidity regulation. Second, at low levels, the liquidity requirement becomes non-binding as the self-insurance motive dominates then, reducing its marginal welfare benefit to zero. In contrast, the marginal benefits of the capital requirement are especially large at low levels of that requirement (see Fig. E2 in the Appendix). More broadly, the marginal welfare benefits are quite nonlinear, rising sharply as requirements are pushed to low levels (except, for the liquidity requirement, once it becomes non-binding). This reflects a similar pattern for the bank failure rate (Fig. E3). In contrast, the marginal welfare costs, which are increasing in each requirement, appear nearly linear.

Taking into account costs and benefits, the post-crisis Basel III reforms to capital and liquidity requirements have resulted in a considerable net increase in welfare, amounting to 0.21 percent in consumption-equivalent terms. This is illustrated by Figure 3, which shows the level of welfare as a function of the two requirements. The nonlinearity of benefits is also clearly visible in this figure: While welfare is relatively flat at high requirements, excessively low capital requirements result in steep welfare losses.

Turning to optimal regulation, we find that the welfare-maximizing levels of the capital and liquidity requirements are close to Basel III levels. Specifically, for the baseline calibration, a capital requirement of 13 percent of risk-weighted assets and a liquidity requirement of 18 percent of deposits maximize welfare. For comparison, the aggregate risk-based tier 1 ratio averages about 13 percent over the Basel III period, and the observed liquidity ratio is about 17 percent for the same period.[60] Of course, model-implied optimal requirements depend on the calibration, and the choice of calibration targets is not equally straightforward for all parameters. We conduct the following sensitivity analysis.

- Accounting for the various estimates of non-interest cost of servicing deposits presented in section 5 has a negligible impact on the optimal capital requirement but yields a range of 18 to 21 percent for the liquidity requirement.
- Using alternative values for the resolution costs that capture existing estimates of the output losses associated with financial crises (BCBS (2010)) results in moderately higher optimal requirements: 15-16 percent and 21-24 percent for the capital and liquidity requirement, respectively.
- Using alternative estimates for the bank failure rate from the literature and one we constructed to account for the possibility that extraordinary government support suppressed the failure rate during the GFC, we obtain 12-13 percent for the capital requirement and 17-18 percent for the liquidity requirement.[61]

Thus, overall, we find a range of 12 to 16 percent for the capital requirement and 17 to 24 for the liquidity requirement.

Finally, we investigate the importance of the leverage and risk-shifting distortions (τ^E and τ^L in the planner's problem) by setting the deadweight resolution costs to zero ($\psi_{Sol} = \psi_{Liq} = 0$).[62] As expected, this reduces optimal requirements. However, a significant role for regulation remains, with an implied optimal capital requirement of 5 percent and an optimal liquidity requirement of 12 percent. Thus, the distortions account for roughly half of the optimal levels of the requirements.

[60] Due to the stress capital buffer and the G-SIB surcharge, tier 1 *required* ratios vary by bank. For the G-SIBs, the largest banks which hold most bank assets, the size-weighted-average tier 1 requirement (including regulatory buffers and the surcharge) was 12.1 percent at end-2025. As mentioned, the LCR rule depends on more detailed balance sheet information than can be captured in a tractable model, and we focus on its implied value for the ratio of liquid assets to short-term liabilities; see also footnote 38.

[61] Accounting for extraordinary government support (by including TARP's Capital Purchase Program recipients) also lowers estimated resolution costs, since the program did not lead to losses to the Deposit Insurance Fund or taxpayers. As a result, it turns out to have little *net* effect on optimal requirements relative to the baseline calibration.

[62] The wedge $\tau^{\bar{w}} = 0$ in the calibration and at optimal requirements.

7 Conclusion

This paper has presented a framework for measuring the welfare costs of bank liquidity and capital requirements. While such requirements have important financial stability benefits, they also entail social costs because they reduce banks' ability to create net liquidity in equilibrium, which in turns impacts investment and economic activity. The cost of the capital requirement scales with the convenience yield on bank deposits, net of the noninterest costs of servicing those deposits. The cost of the liquidity requirement scales with the *difference* between the convenience yields on government bonds and bank deposits (again, net off their noninterest costs). Using U.S. data, the welfare cost of a 10 percent liquidity requirement is found to be equivalent to a permanent loss in consumption of about 0.02% – a modest impact. Even according to conservative estimates, the cost of a 10 percentage point increase in capital requirements is roughly ten times as large. At the same time, based on the calibration and numerical solution, the benefits of the capital requirement are also found to be larger, and welfare-maximizing requirements are close to Basel III levels. Overall, the model suggests an intuitive division of labor: liquidity regulation should address liquidity risk, and capital regulation should address credit risk.

References

Baker, Malcolm, and Jeffrey Wurgler, 2015. "Do Strict Capital Requirements Raise the Cost of Capital? Bank Regulation, Capital Structure, and the Low Risk Anomaly," *American Economic Review*, 105(5), 315-320.

BCBS (Basel Committee on Banking Supervision), 2010. "An assessment of the long-term economic impact of stronger capital and liquidity requirements," (L.E.I. report), Bank for International Settlements, Basel, August.

Begenau, Juliane, 2020. "Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model," *Journal of Financial Economics*, 136(32), 355-378.

Begenau, Juliane, and Tim Landvoigt, 2022. "Regulation in a Quantitative Model of the Modern Banking System," *The Review of Economic Studies*, 89(4), 1748–1784.

Calomiris, Charles, Florian Heider and Marie Hoerova, 2015. "A Theory of Bank Liquidity Requirements," Columbia Business School Research Paper no. 14-39, Columbia University.

Canzoneri, Matthew, Behzad Diba, Luca Guerrieri, and Arsenii Mishin, 2020. "Optimal Dynamic Capital Requirements and Implementable Capital Buffer Rules," Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System (No. 2020-056).

Carlson, Mark, Burcu Duygan-Bump, and William R. Nelson, 2015. "Why Do We Need Both Liquidity Regulations and a Lender of Last Resort? A Perspective from Federal Reserve Lending during the 2007-09 U.S. Financial Crisis," Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System (No. 2015-011).

Chetty, Raj. 2009. "Sufficient statistics for welfare analysis: a bridge between structural and reduced-form methods," *Annual Review of Economics*, 1, 451-488.

Clerc, Laurent, Alexis Derviz, Caterina Mendicino, Stephane Moyen, Kalin Nikolov, Livio Stracca, Javier Suarez, and Alexandros P. Vardoulakis, 2015. "Capital Regulation in a Macroeconomic Model with Three Layers of Default," *International Journal of Central Banking*, 11, 9-64.

Cochrane, John, 2014. "Toward a run-free financial system," in Martin Neil Baily, John B. Taylor, eds., *Across the Great Divide: New Perspectives on the Financial Crisis* (Hoover Press).

Corbae, Dean, and Pablo D'Erasmo, 2021. "Capital Buffers in a Quantitative Model of Banking Industry Dynamics," *Econometrica*, 89(6), 2975-3023.

Correia, Sergio, Stephan Luck, and Emil Verner, 2026. "Failing Banks," *The Quarterly Journal of Economics*, 141(1), 147-204.

Covas, Francisco, and John C. Driscoll, 2014. "Bank Liquidity and Capital Regulation in General Equilibrium," Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System (No. 2014-85).

Dávila, Eduardo, 2019. "Using Elasticities to Derive Optimal Bankruptcy Exemptions," *The Review of Economic Studies*, 87(2), 870-913.

Dávila, Eduardo, and Itay Goldstein, 2023, Optimal Deposit Insurance, *Journal of Political Economy*, 131, 1676-1730.

Davydiuk, Tetiana, 2017. "Dynamic Bank Capital Requirements," working paper, Carnegie Mellon University.

De Nicolò, Gianni, Andrea Gamba and Marcella Lucchetta, 2014. "Microprudential Regulation in a Dynamic Model of Banking," *The Review of Financial Studies*, 27(7), 2097-2138.

Diamond, Douglas and Philip Dybvig, 1983. "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401-419.

Diamond, Douglas W., and Anil K. Kashyap, 2016. "Liquidity Requirements, Liquidity Choice, and Financial Stability," *Handbook of Macroeconomics*, 2, 2263-2303.

Elenev, Vadim, Tim Landvoigt, and Stijn Van Nieuwerburgh, 2021. "A Macroeconomic Model with Financially Constrained Producers and Intermediaries," *Econometrica*, 89(3), 1361-1418.

Ennis, Huberto M., and Alexander L. Wolman, 2015. "Large Excess Reserves in the United States: A View from the Cross-Section of Banks," *International Journal of Central Banking*, 11(1), 251-289.

Farhi, Emmanuel, and Jean Tirole. 2012. "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 102(1), 60-93.

Feenstra, Robert, 1986. "Functional Equivalence between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics,* 17, 73-85.

Flannery, Mark J, and Sorin M. Sorescu, 1996. "Evidence of Bank Market Discipline in Subordinated Debenture Yields: 1983-1991," *Journal of Finance*, 51, 1347-1377.

Friedman, Milton, 1960. *A Program for Monetary Stability* (Fordham University Press).

Gorton, Gary, Toomas Laarits, and Tyler Muir, 2022. "Mobile Collateral versus Immobile Collateral," *Journal of Money, Credit and Banking*, 54(6), 1673-1703.

Gorton, Gary, and Andrew Winton, 2017. "Liquidity Provision, Bank Capital, and the Macroeconomy," *Journal of Money, Credit and Banking*, 49(1), 5-37.

Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein, 2015. "A Comparative-Advantage Approach to Government Debt Maturity," *Journal of Finance,* 70(4), 1683-1722.

Hanson, Samuel G., Andrei Schleifer, Jeremy C. Stein, and Robert W. Vishny, 2015. "Banks as patient fixed income investors," *Journal of Financial Economics*, 117(3), 449-469.

Jermann, Urban, and Haotian Xiang, 2024. "Rules versus Discretion in Capital Regulation," working paper, University of Pennsylvania.

Kara, Gazi I., and S. Mehmet Ozsoy, 2019. "Bank Regulation Under Fire Sale Externalities," *The Review of Financial Studies*, 33(6), 2554–2584.

Kashyap, Anil K., Dimitrios P. Tsomocos, and Alexandros P. Vardoulakis, 2024. "Optimal Bank Regulation In the Presence of Credit and Run-Risk," *Journal of Political Economy*, 132(3), 772-823.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2012. "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120(2), 233-267.

Lucas, Robert E., Jr., 2000. "Inflation and Welfare," *Econometrica*, 68(2).

Malherbe, Frederic, 2020. "Optimal Capital Requirements over the Business and Financial Cycles," *American Economic Journal: Macroeconomics*, 12(3),139-74.

Martinez-Miera, David, and Javier Suarez, 2014. "Banks' Endogenous Systemic Risk Taking," CEPR Discussion Paper 9134, Center for Economic and Policy Research.

Mendoza, Enrique G., and Sergio Villalvazo, 2020. "*FiPIt*: A simple, fast global method for solving models with two endogenous states and occasionally binding constraints," *Review of Economic Dynamics*, 37, 81-102.

Nguyen, Thien, 2015. "Bank Capital Requirements: A Quantitative Analysis," Fisher College of Business Working Paper no. 2015-03-14, Ohio State University.

Ordoñez, Guillermo, 2018. "Sustainable Shadow Banking," *American Economic Journal: Macroeconomics*, 10(1), 1-25.

Romer, Christina D., and David H. Romer, 2017. "New Evidence on the Aftermath of Financial Crises in Advanced Countries," *American Economic Review*, 107(10), 3072–3118.

Sidrauski, Miguel, 1967. "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review*, 57(2), 534-544.

Sraer, David, and David Thesmar, 2018. "A Sufficient Statistics Approach for Aggregating Firm-Level Experiments," Working Paper 24208, National Bureau of Economic Research.

Stein, Jeremy C., 2012. "Monetary Policy as Financial Stability Regulation." *The Quarterly Journal of Economics* 127, 57-95.

Van den Heuvel, Skander J., 2006. "The Welfare Cost of Bank Capital Requirements," Rodney L. White Center Working Paper no. 15-06, University of Pennsylvania. Available at https://rodneywhitecenter.wharton.upenn.edu/wp-content/uploads/2014/04/0615.pdf

Van den Heuvel, Skander J., 2008. "The Welfare Cost of Bank Capital Requirements," *Journal of Monetary Economics*, 55(2), 298-320.

Vives, X., 2014. "Strategic Complementarity, Fragility, and Regulation," *The Review of Financial Studies*, 27(12), 3547-3592.

Woodford, Michael, 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy* (Princeton University Press).

Appendix A. The Bank's Problem in the Baseline Model

After scaling the resulting problem in (4) by R^E and using the balance sheet identity to substitute out B, the Lagrangian and first-order conditions (FOCs) are:[63]

$$\mathcal{L} = R^L L + R^B(E + D - L) - R^D D - g(D, L) - R^E E + \Lambda[E + (1-\lambda)D - L] + \chi[E - \gamma L]$$

$$\begin{aligned} (L) \quad R^L &= R^B + g_L + \Lambda + \gamma\chi \\ (E) \quad R^E &= R^B + \Lambda + \chi \\ (D) \quad R^D + g_D &= R^B + (1-\lambda)\Lambda \end{aligned}$$

The complementary slackness conditions are: $\chi[E - \gamma L] = 0,\ \chi \geq 0,\ \Lambda[E + (1-\lambda)D - L] = 0,\ \Lambda \geq 0$. Note that

$$R^B + \Lambda = R^D + g_D + \lambda\Lambda = R^L - g_L - \gamma\chi = R^E - \chi \tag{39}$$

Since the Kuhn-Tucker multipliers must be nonnegative, a finite solution requires the ranking of returns shown in (6) in the proposition: $R^B \leq R^D + g_D \leq R^L - g_L \leq R^E$.

From FOC (D), $\Lambda = \frac{1}{1-\lambda}(R^D + g_D - R^B)$. Hence, the liquidity requirement binds if and only if $R^D + g_D > R^B$. In addition, from (39), $\chi = \frac{1}{1-\gamma}(R^E - (R^L - g_L))$. Hence, the capital requirement binds if and only if $R^E > R^L - g_L$. Further, (39) implies that $R^L - g_L - \gamma\chi = \gamma\{R^E - \chi\} + (1-\gamma)\{R^D + g_D + \lambda\Lambda\}$. Rearranging and using the expression for Λ as well as the definition of $\tilde{R}^D(\lambda)$ (see (5)) yields the (marginal) zero-profit condition:

$$R^L - g_L(D, L) = \gamma R^E + (1-\gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L)\} \tag{40}$$

which is (7) in the proposition. This implies that $R^L - g_L - R^E = (1-\gamma)[\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D\} - R^E]$, yielding the equivalent, alternative condition for a (non)binding capital requirement:

$$R^L - g_L(D, L) < (=)\ R^E \iff \tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L) < (=)\ R^E$$

[63] For brevity, the arguments of $g(D, L)$ and its partial derivatives are often suppressed.

Zero profits follow from

$$\begin{aligned}
R^E \pi^B &= R^L L + R^B B - R^D D - g(D, L) - R^E E \\
&= R^L L + R^B(E + D - L) - R^D D - (D g_D(D, L) + L g_L(D, L)) - R^E E \\
&= [R^L - g_L(D, L) - R^B]L - [R^D + g_D(D, L) - R^B]D - [R^E - R^B]E \\
&= [\Lambda + \gamma\chi]L - [(1-\lambda)\Lambda]D - [\Lambda + \chi]E \\
&= -\chi(E - \gamma L) - \Lambda[E + (1-\lambda)D - L] = 0
\end{aligned}$$

where the steps follow from Euler's theorem, the first-order conditions and the complementary slackness conditions, in that order.

This concludes the proof of proposition 1, except for lemma 1, which follows below.

Statement and proof of lemma 1

The following lemma enumerates when each of the two regulatory requirements is binding or slack as a function of variables that the bank takes as given only (that is, independent of D and L).[64] It also shows that all four cases are possible.

Lemma 1 *Let $\rho \equiv (1-\gamma)/(1-\lambda)$. For a finite solution to the bank's problem in (4), four cases are possible:*

1. *If $R^B = R^E$, then both regulatory requirements are slack, and all relations in (6) hold with equality.*
2. *If $R^B < R^E$, $R^B < R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) \geq R^E$, then the liquidity requirement binds, so $B = \lambda D$, the capital requirement is slack, and*

$$R^B < R^D + g_D(D, L) < R^L - g_L(D, L) = R^E = \tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L)$$

3. *If $R^B < R^E$, $R^B \geq R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) < R^E$, then the liquidity requirement is slack, the capital requirement binds, so $E = \gamma L$, and*

$$R^B = R^D + g_D(D, L) < R^L - g_L(D, L) = \gamma R^E + (1-\gamma) R^B < R^E$$

[64] The issue is that the conditions derived so far – that is, $R^D + g_D(D, L) > R^B$ for a biding liquidity requirement and $R^E > R^L - g_L(D, L)$ for a binding capital requirement – still depend on two decision variables, D and L, when $g \neq 0$, so that the characterization of the solution is not complete.

4. *If* $R^B < R^E$, $R^B < R^D + g_D(\rho,1)$ *and* $R^L - g_L(\rho,1) < R^E$, *then both regulatory requirements bind, so* $B = \lambda D$ *and* $E = \gamma L$, *all inequalities in (6) are strict, and*

$$R^L = \gamma R^E + (1-\gamma)\tilde{R}^D(\lambda) + g(\rho,1)$$

Proof The solution requires $\chi \geq 0$ and $\Lambda \geq 0$.

Case 1. Nonbinding constraints ($\chi = 0$ and $\Lambda = 0$) From (39), $R^B = R^D + g_D = R^L - g_L = R^E$. Note that this case requires $R^D \leq R^B = R^E \leq R^L$. With the partial derivatives of g homogenous of degree 0, the ratio D/L is determined by $g_D(D/L,1) = R^B - R^D$ and by $g_L(D/L,1) = R^L - R^B$. A solution requires that the configuration of returns is such that both equations imply the same value for D/L.

Case 2. Only liquidity requirement binds ($\chi = 0$ and $\Lambda > 0$) From the FOCs, $R^L - g_L(D,L) = R^E$. From this and (40) it follows that

$$R^L - g_L(D,L) = R^E = \tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D,L)$$

Again, we have two equations that each pin down the ratio D/L. This case requires that $R^B < R^D + g_D(D,L) < R^L - g_L(D,L) = R^E$ and in particular $R^B < R^E$. Recall that ρ is defined as the value of D/L when both regulatory constraints are binding: $\rho = \frac{1-\gamma}{1-\lambda}$. Due to the nonbinding capital requirement here, $D/L \leq \rho$. Hence, as g is convex, $g_D(D,L) = g_D(D/L,1) \leq g_D(\rho,1)$ and $g_L(D,L) = g_L(D/L,1) \geq g_L(\rho,1)$. Thus, $R^B < R^D + g_D(\rho,1)$ and $R^L \geq R^E + g_L(\rho,1)$.

Case 3. Only capital requirement binds ($\chi > 0$ and $\Lambda = 0$) From the FOCs, $R^B = R^D + g_D(D,L)$ and

$$R^L - g_L(D,L) = \gamma R^E + (1-\gamma)(R^D + g_D(D,L)) = \gamma R^E + (1-\gamma)R^B$$

This case requires that $R^B = R^D + g_D(D,L) < R^L - g_L(D,L) < R^E$. Due to the nonbinding liquidity requirement, $D/L \geq \rho$, so $g_D(D,L) \geq g_D(\rho,1)$ and $g_L(D,L) \leq g_L(\rho,1)$. Hence, $R^B \geq R^D + g_D(\rho,1)$ and $R^L < R^E + g_L(\rho,1)$. Also, $R^D + g_D(\rho,1) < R^E$.

Case 4. Both requirements bind ($\chi > 0$ and $\Lambda > 0$) In this case, $D/L = \rho$, so (40) implies $R^L - g_L(\rho,1) = \gamma R^E + (1-\gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(\rho,1)\}$. Using Euler's theorem and $\rho = (1-\gamma)/(1-\lambda)$, we have $R^L = \gamma R^E + (1-\gamma)\tilde{R}^D(\lambda) + g(\rho,1)$, as stated. $g(\rho,1)$ is the total noninterest cost of making one unit of loans and servicing ρ units of deposits. With $\chi > 0$ and $\Lambda > 0$, the inequalities in the ranking of returns (6) are all strict, and, with $D/L = \rho$, $R^B < R^D + g_D(\rho, 1) < R^L - g_L(\rho, 1) < R^E$.

Collecting these results concludes the proof of the lemma and proposition 1.[65]

Appendix B. Equilibrium and Planner's Problem in the Baseline Model

Competitive equilibrium

Combining (12) and (13) with (1), (2), (3), (5), (8), (9) (10), the bank balance sheet identity, and proposition 1, the resulting equilibrium allocation can be characterized in terms of a dynamic system in (K_t, c_t) with R_t^E, L_t, d_t, and b_t as auxiliary variables:

$$K_{t+1} = F(K_t, 1) + (1 - \delta)K_t - c_t - g(d_t, L_t) - \varphi(K_t - L_t) \tag{41}$$

$$R_t^E = (\beta u_c(c_t, d_t, b_t)/u_c(c_{t-1}, d_{t-1}, b_{t-1}))^{-1} \tag{42}$$

$$F_K(K_t, 1) + 1 - \delta = R_t^L = R_t^E - \Delta_L(c\,, d_t, b_t, L_t) \tag{43}$$

with the spread $\Delta_L = R^E - R^L$ defined by:[66]

$$\Delta_L(c_t\,, d_t, b_t, L_t) \equiv \rho\left(\frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - g_D(d_t, L_t) - \lambda\frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)}\right) - g_L(d_t, L_t) \tag{44}$$

where $\rho \equiv (1 - \gamma)/(1 - \lambda)$ and where L_t, d_t, and b_t are jointly determined by the following equilibrium versions of the firms' and banks' complementary slackness conditions:

$$\text{If } \Delta_L(c_t, d_t, b_t, K_t) > -\varphi, \text{ then } L_t = K_t; \text{ else } \Delta_L(c_t, d_t, b_t, L_t) = -\varphi \tag{45}$$

$$\begin{gathered}\text{If } \Delta_L(c_t, d_t, b_t, L_t) > -g_L(d_t, L_t), \text{ then } d_t = (1 - \gamma)L_t + \bar{B} - b_t; \\ \text{else } \Delta_L(c_t, d_t, b_t, L_t) = -g_L(d_t, L_t) \text{ and } (d_t - \bar{B} + b_t)/(1 - \gamma) \le L_t \le K_t\end{gathered} \tag{46}$$

[65] The case $R^B < R^E$, $R^B \geq R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) \geq R^E$ is missing from the lemma because it is incompatible with a finite solution. It is straightforward to show that feasible choices $B = \lambda D$ and $E = \gamma L$ result in strictly positive profits and an optimal scale that is infinite in this case.

[66] The expression for Δ_L follows from (7) and (5): $R^E - R^L = (1 - \gamma)(R^E - \tilde{R}^D(\lambda)) - \rho g_D(D, L) - g_L(D, L) = \rho(R^E - R^D) - \lambda\rho(R^E - R^B) - \rho g_D(D, L) - g_L(D, L)$.

$$\text{If } \Delta_B(c_t, d_t, b_t, L_t) > 0, \text{ then } \bar{B} - b_t = \lambda d_t; \text{ else } \Delta_B(c_t, d_t, b_t, L_t) = 0 \tag{47}$$

with the wedge $\Delta_B = R^D + g_D - R^B$ given by:

$$\Delta_B(c_t, d_t, b_t, L_t) \equiv \frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - \frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} + g_D(d_t, L_t) \tag{48}$$

Discussion and derived properties The first and second equations restate, respectively, the social resource constraint and the household's intertemporal optimality condition (1), which determines the required return on (riskless) equity. The marginal product of capital is equal to the lending rate, which can be below the equity return, as acknowledged by (43). The spread between the return on equity and then lending rate, Δ_L, is given by (44), which is the equilibrium version of the bank's zero profit condition (7).

From (44) and (46), note that the capital requirement binds if and only if

$$u_d(c_t, d_t, b_t) - g_D(d_t, L_t)u_c(c_t, d_t, b_t) > \lambda u_b(c_t, d_t, b_t)$$

and, from (47) and (48), the liquidity requirement binds if and only if

$$u_b(c_t, d_t, b_t) > u_d(c_t, d_t, b_t) - g_D(d_t, L_t)u_c(c_t, d_t, b_t)$$

A *pure bank finance* equilibrium obtains when $R^L < R^E + \varphi$, which implies $L = K$ (see (10)). In equilibrium, this requires $\Delta_L(c, d, b, K) + \varphi > 0$, as stated in (45). If $\varphi \leq g_L(d, K)$, then such an equilibrium must have a binding capital requirement; otherwise, it could be binding or slack, depending on (46) (see also proposition 1). Similarly, the liquidity requirement can be binding or slack according to (47). Specifically, all four cases listed in lemma 1 are theoretically possible as part of a pure bank finance equilibrium:[67]

1. Pure bank finance with a binding capital and liquidity requirements is an equilibrium if, for $d = \rho K$, $b = \bar{B} - \lambda \rho K$, and $L = K$, we have $\Delta_L > \max(-\varphi, -g_L)$ and $\Delta_B > 0$.

2. Pure bank finance with a slack liquidity requirement and binding capital requirement is an equilibrium if there is a level of household bond holdings b such that, for $d = (1-\gamma)K + \bar{B} - b$ and $L = K$, we have $\Delta_B = 0$ and $\Delta_L > \max(-\varphi, -g_L)$.

3. Pure bank finance with a slack capital requirement and binding liquidity requirement is an equilibrium if there is a level of deposits d such that, for $b = \bar{B} - \lambda d$ and $L = K$, we have $\Delta_L = -g_L$, $\Delta_L > -\varphi$, and $\Delta_B > 0$. This case requires $\varphi > g_L$.

[67] The arguments of $\Delta_L(c, d, b, L)$, $\Delta_B(c, d, b, L)$ and $g_L(d, L)$ are suppressed for brevity.

4. Pure bank finance with both requirements slack is an equilibrium if there are levels of bond holding b and deposits d such that, with $L = K$, we have $\Delta_L = -g_L$, $\Delta_L > -\varphi$, and $\Delta_B = 0$. Again, this case requires $\varphi > g_L$.

When $\Delta_L(c, d, b, K) < -\varphi$, pure bank finance is not an equilibrium. Instead, the equilibrium is characterized by *mixed finance* ($L < K$), with $R^E = R^L - \varphi$ and the relative size of the banking sector determined endogenously by $\Delta_L(c, d, b, L) = -\varphi$ (see (45)). Note that a mixed finance equilibrium requires that $\varphi \leq g_L(d, L)$ (as $-\varphi = \Delta_L \geq -g_L$). Thus, NBFI lending may occur in equilibrium only if the noninterest cost of nonbank finance is no greater than the marginal noninterest cost of bank credit. This reflects the fact that nonbanks do not have the funding advantage of deposits. In a mixed finance equilibrium, the capital and liquidity requirements can each be slack or binding, according to conditions (46) and (47), respectively. However, as shown below, the capital requirement can only be slack in the knife-edge case of equal costs: $\varphi = g_L$. The four possible cases for a mixed finance equilibrium are as follows:

1. Mixed finance with binding capital and liquidity requirements is an equilibrium if there is a level of loans L such that, for $d = \rho L$ and $b = \bar{B} - \lambda\rho L$, we have $\Delta_L = -\varphi$, $\Delta_L > -g_L$, and $\Delta_B > 0$. Note that this case requires $g_L > \varphi$.

2. Mixed finance with a slack liquidity requirement and binding capital requirement is an equilibrium if there are levels of L and b such that, for $d = (1-\gamma)L + \bar{B} - b$, we have $\Delta_L = -\varphi$, $\Delta_L > -g_L$, and $\Delta_B = 0$. This case also requires $g_L > \varphi$.

3. Mixed finance with a slack capital requirement and binding liquidity requirement is an equilibrium if there are levels of L and d such that, for $b = \bar{B} - \lambda d$, we have $\Delta_L = -\varphi = -g_L$, and $\Delta_B > 0$. Note that this case requires $g_L = \varphi$.

4. Mixed finance with both requirements slack is an equilibrium if there are levels of L, b, and d such that $\Delta_L = -\varphi = -g_L(d, L)$ and $\Delta_B = 0$. This case also requires $g_L = \varphi$.

It is apparent from the above that the relative magnitudes of the noninterest costs of credit influence the characteristics of the equilibrium. Summing up, a mixed finance equilibrium can only occur if $\varphi \leq g_L$, whereas a pure bank finance is possible for any value of φ, e.g., if the demand for bank deposits is sufficiently strong. Additionally, the capital requirement always binds if $\varphi < g_L$; otherwise, it could be binding or slack (depending on other primitives).

Equivalence of the planner's problem and the competitive equilibrium

The Lagrangian and first-order conditions to the constrained planner's problem in (16) are:

$$\begin{aligned}\mathcal{L} = \max_{\{c_t,d_t,b_t,L_t,K_{t+1}\}_{t=0}^{\infty}} \sum\nolimits_{t=0}^{\infty} \beta^t \{ & u(c_t,d_t,b_t) + \omega_t^{sp}[F(K_t,1) + (1-\delta)K_t - c_t - K_{t+1} \\ & -g(d_t,L_t) - \varphi(K_t - L_t)] + \Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] \\ & +\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] + \mu_t^{sp}[K_t - L_t]\}\end{aligned}$$

$$\begin{aligned}(c) \quad u_c(c_t,d_t,b_t) &= \omega_t^{sp} \\ (d) \quad u_d(c_t,d_t,b_t) &= \omega_t^{sp} g_D(d_t,L_t) + \Lambda_t^{sp}\lambda + \chi_t^{sp} \\ (b) \quad u_b(c_t,d_t,b_t) &= \Lambda_t^{sp} + \chi_t^{sp} \\ (L) \quad \chi_t^{sp}(1-\gamma) &= \omega_t^{sp}(g_L(d_t,L_t) - \varphi) + \mu_t^{sp} \\ (K) \quad \omega_t^{sp}[F_K(K_t,1) + 1 - \delta - \varphi] &= \beta^{-1}\omega_{t-1}^{sp} - \mu_t^{sp}\end{aligned}$$

with $\Lambda_t^{sp} \geq 0$, $\Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] = 0$, $\chi_t^{sp} \geq 0$, $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.

Subtract λ times the first-order condition with respect to bonds (FOC (b)) from FOC (d) to obtain $u_d - \lambda\, u_b = (1-\lambda)\chi_t^{sp} + \omega_t^{sp} g_D$ (omitting arguments for brevity). Solving for χ_t^{sp} and inserting the result into FOC (L) and using FOC (c) yields:

$$\frac{\mu_t^{sp}}{\omega_t^{sp}} = \rho\left(\frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - \lambda\, \frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - g_D(d_t,L_t)\right) - g_L(d_t,L_t) + \varphi = \Delta_L(c_t,d_t,b_t,L_t) + \varphi$$

(Recall that $\rho = (1-\gamma)/(1-\lambda)$.) Inserting this into FOC (K) and using FOC (c) yields:

$$F_K(K_t,1) + 1 - \delta = \beta^{-1}\frac{u_c(c_{t-1},d_{t-1},b_{t-1})}{u_c(c_t,d_t,b_t)} - \Delta_L(c_t,b_t,d_t,L_t)$$

This replicates equations (42), (43) and (44) in the characterization of the decentralized equilibrium. Furthermore, (45) follows from $\Delta_L(c_t,d_t,b_t,L_t) + \varphi = \mu_t^{sp}/\omega_t^{sp}$, $\omega_t^{sp} = u_c > 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.

Since $\chi_t^{sp}(1-\gamma) = \mu_t^{sp} + \omega_t^{sp}(g_L(d_t,L_t) - \varphi)$ (from FOC(L)) and $\mu_t^{sp} = \omega_t^{sp}(\Delta_L(c_t,d_t,b_t,L_t) + \varphi)$, $\chi_t^{sp}(1-\gamma) = \omega_t^{sp}(\Delta_L(c_t,d_t,b_t,L_t) + g_L(d_t,L_t))$. As $\omega_t^{sp} > 0$, $\chi_t^{sp} \geq 0$ and $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, it follows that $\chi_t^{sp} > 0$ and $d_t = (1-\gamma)L_t + \bar{B} - b_t$ if $\Delta_L(c_t,d_t,b_t,L_t) > -g_L(d_t,L_t)$; otherwise $\chi_t^{sp} = 0$, $d_t \leq (1-\gamma)L_t + \bar{B} - b_t$, and $\Delta_L(c_t,d_t,b_t,L_t) = -g_L(d_t,L_t)$, a result that is equivalent to (46) in the decentralized equilibrium.

Taking the difference between FOC(b) and FOC(d) yields

$$(1-\lambda)\frac{\Lambda_t^{sp}}{\omega_t^{sp}} = \frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - \frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} + g_D(d_t,L_t) \equiv \Delta_B(c_t,d_t,b_t,L_t) \qquad (49)$$

the expression in (48). (47) follows from $\Delta_B(c_t,d_t,b_t,L_t) = (1-\lambda)\Lambda_t^{sp}/\omega_t^{sp}$, $\omega_t^{sp} = u_c > 0$, $\Lambda_t^{sp} \geq 0$, and $\Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] = 0$. Finally, equation (41) in the characterization of the decentralized equilibrium is included as one of the constraints of the planner's problem.

Collecting these results, it is apparent that the allocations of K_t, c_t, b_t, d_t and L_t implied by the planner's problem are identical to those of the decentralized equilibrium summarized in equations (41)-(48). Hence, the constrained social planner's problem replicates the decentralized equilibrium. Moreover, under those conditions, welfare equals $V_0(\theta)$, as defined in (16).

Proof of proposition 2

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising the liquidity requirement λ is:

$$\begin{aligned}\frac{\partial V_0(\theta)}{\partial \lambda} &= -\sum\nolimits_{t=0}^{\infty} \beta^t \Lambda_t^{sp} d_t \\ &= -\sum\nolimits_{t=0}^{\infty} \beta^t \{u_b(c_t,d_t,b_t) - u_d(c_t,d_t,b_t) + u_c(c_t,d_t,b_t) g_D(d_t,L_t)\} \frac{d_t}{1-\lambda}\end{aligned}$$

(see (49)). Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, their equilibrium values can be used. Moreover, in that equilibrium, we have, by taking the difference between the household's first-order conditions (2) and (3),

$$u_b(c_t,d_t,b_t) - u_d(c_t,d_t,b_t) = u_c(c_t,d_t,b_t)(R_t^D - R_t^B)$$

Thus,

$$\frac{\partial V_0(\theta)}{\partial \lambda} = -\sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t,d_t,b_t)(R_t^D + g_D(d_t,L_t) - R_t^B)\} \frac{d_t}{1-\lambda}$$

It is standard to compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu_{Liq})$, which equals, to a first-order approximation, $\sum_{s=0}^{\infty} \beta^s u_c(c_s,d_s,b_s) c_s \nu_{Liq}$. Equating this to the right-hand side of the previous equation yields for the marginal welfare cost in relative consumption equivalents:

$$\nu_{Liq} = \sum\nolimits_{t=0}^{\infty} \varpi_t \frac{d_t}{c_t} \left\{ R_t^D - R_t^B + g_D(d_t, L_t) \right\} \frac{1}{1-\lambda} \tag{50}$$

where ϖ_t are a series of positive weights that sum to one:

$$\varpi_t \equiv \frac{\beta^t u_c(c_t, d_t, b_t) c_t}{\sum_{s=0}^{\infty} \beta^s u_c(c_s, d_s, b_s) c_s} \tag{51}$$

Under the assumption that the economy is in steady state in period 0, all variables in (50) are constant over time, except for the weights, which sum to one. Hence, in that case,

$$\nu_{Liq} = \frac{d_0}{c_0} \left\{ R_0^D - R_0^B + g_D(d_0, L_0) \right\} \frac{1}{1-\lambda}$$

as claimed.

Proof of proposition 3

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising γ is:

$$\begin{aligned} \frac{\partial V_0(\theta)}{\partial \gamma} &= -\sum\nolimits_{t=0}^{\infty} \beta^t \chi_t^{sp} L_t \\ &= -\sum\nolimits_{t=0}^{\infty} \beta^t \{ u_d(c_t, d_t, b_t) - \lambda\ u_b(c_t, d_t, b_t) - u_c(c_t, d_t, b_t) g_D(d_t, L_t) \} \frac{L_t}{1-\lambda} \end{aligned}$$

where the second equality follows from the planner's first-order conditions for bonds, deposits and consumption. Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, their equilibrium values can be used. Moreover, in that equilibrium, we have, from the household's first-order conditions (2) and (3),

$$\begin{aligned} u_d(c_t, d_t, b_t) - \lambda u_b(c_t, d_t, b_t) &= u_c(c_t, d_t, b_t) \{ R_t^E - R_t^D - \lambda (R_t^E - R_t^B) \} \\ &= u_c(c_t, d_t, b_t) \{ (1-\lambda)(R_t^E - R_t^D) - \lambda (R_t^D - R_t^B) \} \\ &= u_c(c_t, d_t, b_t)(1-\lambda)(R_t^E - \tilde{R}_t^D(\lambda)) \end{aligned}$$

Hence,

$$\frac{\partial V_0(\theta)}{\partial \gamma} = -\sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t) \{ R_t^E - \tilde{R}_t^D(\lambda) - \frac{g_D(d_t, L_t)}{1-\lambda} \} L_t$$

Again, it is standard to compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu_{Cap})$, which equals, to a first-order approximation, $\sum_{s=0}^{\infty}\beta^s u_c(c_s, d_s, b_s)c_s\nu_{Cap}$. Equating this to the right-hand side of the previous equation yields for the marginal welfare cost in relative consumption equivalents:

$$\nu_{Cap} = \sum\nolimits_{t=0}^{\infty} \varpi_t \frac{L_t}{c_t}\left\{R_t^E - \tilde{R}_t^D(\lambda) - \frac{g_D(d_t, L_t)}{1-\lambda}\right\} \tag{52}$$

where the weights ϖ_t are as defined above in (51).

With the assumption that the economy is in steady state in period 0,

$$\nu_{Cap} = \frac{L_0}{c_0}\left\{R_0^E - \tilde{R}_0^D(\lambda) - \frac{g_D(d_0, L_0)}{1-\lambda}\right\}$$

as claimed.

Appendix C. Expanded Model Derivations

C.1. Firms

Firms maximize expected profits. After the realization of ε_t, realized profits (net cash flow) accruing to the owners of the firm are:

$$\pi_t^F(\varepsilon_t) = F(K_t, H_t) + \varepsilon_t K_t + (1-\delta)K_t - W_t H_t - R_t^L(\varepsilon_t)K_t$$

(For now, it is easiest to imagine a state-contingent loan rate, but we will later see that this is equivalent to a standard contract with a non-state-contingent loan rate and firm default.) Maximizing expected profits $\mathbb{E}[\pi_t^F(\varepsilon_t)]$ yields the first-order conditions:

$$(H) \quad F_H(K_t, H_t) = W_t \tag{53}$$

$$(K) \quad F_K(K_t, H_t) + 1 - \delta = \mathbb{E}[R_t^L(\varepsilon_t)] \equiv R_t^L \tag{54}$$

as claimed in the main text.

State-contingent loan repayment rate Limited liability of the owners implies that the net cash flow to shareholders $\pi_t^F(\varepsilon_t) \geq 0$ in all states. Suppose that for some value(s) of ε_t with strictly positive probability, $\pi_t^F(\varepsilon_t) > 0$. Since no owner's capital supplied and there is free entry of firms, this would lead to more entry of firms and/or infinite loan demand of existing firms. (Recall that F exhibits constant returns to scale.) In equilibrium, $R_t^L(\varepsilon_t)$ would adjust upward such that

$$\pi_t^F(\varepsilon_t) = 0$$

in each state. This means that $R_t^L(\varepsilon_t)$ must satisfy

$$R_t^L(\varepsilon_t) = F(1, h_t) + \varepsilon_t + 1 - \delta - W_t h_t$$

where $h_t \equiv H_t/K_t$, which is determined by the first-order condition for labor (53), which is equivalent to $F_H(1, h_t) = W_t$ (as the derivatives of F are homogenous of degree 0). Since $R_t^L \equiv \mathbb{E}[R_t^L(\varepsilon_t)]$ and $\mathbb{E}[\varepsilon_t] = 0$, we have

$$R_t^L(\varepsilon) = R_t^L + \varepsilon_t$$

As banks are assumed to specialize their lending in a specific sector and ε_t is a sectoral shock, this constitutes the lending rate assumed in the subsection on banks, with ε_t being the same shock. Even if banks lend to many firms in the sector, the exposure to this shock remains the same per dollar lent.[68]

Standard loan contract implementation The state contingent loan repayment rate $R_t^L(\varepsilon)$ can be implemented through a standard loan contract with contractual rate $R_t^{L,c}$ set at

$$R_t^{L,c} = R_t^L(\bar{\varepsilon}) = F(1, h_t) + \bar{\varepsilon} + 1 - \delta - W_t h_t = 0$$

where $\bar{\varepsilon}$ is the finite upper bound for ε (recall the assumption of bounded support). (Note that $R_t^{L,c}$ only depends on technology and the wage rate, as W_t pins down h_t.) If $\varepsilon = \bar{\varepsilon}$, then the firm's cash flow is just sufficient to repay the loan without default. If $\varepsilon < \bar{\varepsilon}$, then the firm defaults, the owners receive zero due to limited liability, while the lender receives total gross cash flow (net of wages) $F(K_t, H_t) + \varepsilon_t K_t + (1 - \delta)K_t - W_t H_t$, which equals $R_t^L(\varepsilon)L_t$. Per dollar lent, charge-offs are $R^{L,c} - R^L(\varepsilon) = \bar{\varepsilon} - \varepsilon$.

[68] It can be shown that due to the put option associated with deposit insurance, banks would not want to diversify across sectors even if they could. For the same reason, if ε were firm-specific, banks would opt to lend to a single firm to maximize their exposure to credit risk, if supervisors allowed that.

C.2. Banks

Analysis of bank's problem (proof of proposition 4)

The timing and independence assumptions regarding w and ε imply that expected dividends (the expected value in (22)) equal

$$\mathbb{E}_w[1_{\{B \geq wD\}}]\mathbb{E}_\varepsilon\left[\{(R^L+\varepsilon)L + R^B B - R^D D - g(D,L)\}^+\right]$$

Note that

$$\mathbb{E}_w[1_{\{B \geq wD\}}] = \Pr[w \leq B/D] = F_w(\bar{w})$$

Define the function $H(x)$ as

$$H(x) \equiv \mathbb{E}_\varepsilon\left[\{(x+\varepsilon)\}^+\right]$$

$H(x)$ is a moment of the distribution of ε. Since ε has mean zero and since the mapping $z \longmapsto z^+$ is convex, by Jensen's inequality, $H(x) \geq x^+$. ($H(x)$ is close to 0 when x is far below zero and close to x when x is far above zero.) As $H(x) = \int_{-x}^{\bar{\varepsilon}}(x+\varepsilon)dF_\varepsilon(\varepsilon)$, we have $H'(x) = \int_{-x}^{\bar{\varepsilon}} dF_\varepsilon(\varepsilon) - (x-x)F'_\varepsilon(-x)$ so that

$$H'(x) = 1 - F_\varepsilon(-x) = \Pr[\varepsilon > -x] \tag{55}$$

The following relation will also be useful below. Since $H(x) = \Pr[x+\varepsilon > 0]\mathbb{E}\left[x+\varepsilon \mid x+\varepsilon > 0\right]$ $= \Pr[\varepsilon > -x](x + \mathbb{E}\left[\varepsilon|\varepsilon > -x\right])$,

$$H(x) = H'(x)(x + \mathbb{E}\left[\varepsilon|\varepsilon > -x\right]) \tag{56}$$

Using this notation and the definition of $r_L \equiv R^L + R^B(B/L) - R^D(D/L) - g(D/L,1)$,

$$\mathbb{E}_\varepsilon\left[\{(R^L+\varepsilon)L + R^B B - R^D D - g(D,L)\}^+\right] = H(r_L)L$$

Hence, from (22)

$$\pi^B = \max_{L,B,D,E} F_w(\bar{w})\, H(r_L)L/R^E - E \tag{57}$$

subject to (23), (25), and the constraints in (4). Scale the problem by R^E and use the balance sheet identity to substitute out B to obtain the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= [F_w(\bar{w})H(r_L)L - R^E E] + \Phi_L[R^L L + R^B(E + D - L) - R^D D - g(D, L) - r_L L] \\ &\quad +\Phi_w[E + (1 - \bar{w})D - L] + \Lambda[E + (1 - \lambda)D - L] + \chi[E - \gamma L]\end{aligned}$$

The constraint with multiplier Φ_L contains the definition of r_L and the constraint with multiplier Φ_w contains the definition of $\bar{w}$: $B = \bar{w}D$. Choice variables are $L, B, D, E, \bar{w}, r_L$. The first-order conditions (FOCs) are:[69]

$$\begin{aligned}(L) \quad F_w(\bar{w})H(r_L) + \Phi_L[R^L - R^B - g_L - r_L] - \Phi_w - \Lambda - \gamma\chi &= 0 \\ (E) \quad -R^E + \Phi_L R^B + \Phi_w + \Lambda + \chi &= 0 \\ (D) \quad \Phi_L[R^B - R^D - g_D] + \Phi_w(1 - \bar{w}) + (1 - \lambda)\Lambda &= 0 \\ (r_L) \quad F_w(\bar{w})H'(r_L)L - \Phi_L L &= 0 \\ (\bar{w}) \quad F'_w(\bar{w})H(r_L)L - \Phi_w D &= 0\end{aligned}$$

The complementary slackness conditions are: $\chi[E - \gamma L] = 0,\ \chi \geq 0,\ \Lambda[E + (1 - \lambda)D - L] = 0,\ \Lambda \geq 0$. From FOC (r_L),

$$\Phi_L = F_w(\bar{w})H'(r_L) = F_w(\bar{w})(1 - F_\varepsilon(-r_L)) \in (0, 1] \tag{58}$$

where the second equality follows from (55), and the strict positivity follows from $F_w(0) > 0$ and $F_\varepsilon(-r_L) < 1$, as $F_\varepsilon(-r_L) = 1$ would mean that the bank would fail with probability one, which is inconsistent with the supply of any equity by shareholders, who receive zero return in failure.

Note that Φ_L is the *overall probability of not failing*. It is the product of two terms: $F_w(\bar{w}) = \Pr[w \leq \bar{w}]$ is the probability of surviving liquidity stress and $1 - F_\varepsilon(-r_L) = \Pr[\varepsilon > -r_L]$ is the probability of surviving credit risk. Failure occurs with the complementary probability:

$$p_F = 1 - \Phi_L = 1 - F_w(\bar{w})(1 - F_\varepsilon(-r_L)) \in [0, 1) \tag{59}$$

Thus, the failure probability is completely determined by r_L, $\bar{w}$ and the distributions of ε and w.

It turns out be helpful to work with variables normalized by the survival probability $\Phi_L > 0$. Hat notation will be used to denote variables normalized in this way: $\hat{H}(r_L) =$

[69] For brevity, arguments of $g(D, L)$ and its derivatives are omitted where this does not lead to confusion.

$H(r_L)/\Phi_L$, $\hat{R}^E = R^E/\Phi_L$, $\hat{\Lambda} = \Lambda/\Phi_L$, $\hat{\chi} = \chi/\Phi_L$, $\hat{\Phi}_w = \Phi_w/\Phi_L$. Since $\Phi_L \in (0,1]$, we have $\hat{R}^E \geq R^E$. Dividing the FOCs by Φ_L and rearranging yields

$$\begin{aligned}
(L) \quad R^L - g_L &= R^B + r_L - F_w(\bar{w})\hat{H}(r_L) + \hat{\Lambda} + \gamma\hat{\chi} + \hat{\Phi}_w \\
(E) \quad \hat{R}^E &= R^B + \hat{\Lambda} + \hat{\chi} + \hat{\Phi}_w \\
(D) \quad R^D + g_D &= R^B + (1-\lambda)\hat{\Lambda} + (1-\bar{w})\hat{\Phi}_w \\
(\bar{w}) \quad F_w'(\bar{w})\hat{H}(r_L)L &= \hat{\Phi}_w D
\end{aligned}$$

Note that, using (56) and (58),

$$r_L - F_w(\bar{w})\hat{H}(r_L) = r_L - F_w(\bar{w})H(r_L)/\Phi_L = r_L - (r_L + \mathbb{E}[\varepsilon|\varepsilon > -r_L]) = -\mathbb{E}[\varepsilon|\varepsilon > -r_L]$$

Hence, we can write the FOC with respect to loans as

$$(L) \quad R^{L,d} - g_L = R^B + \hat{\Lambda} + \gamma\hat{\chi} + \hat{\Phi}_w$$

where $R^{L,d}$ is the distorted loan return defined in (26): $R^{L,d} \equiv R^L + \mathbb{E}[\varepsilon|\varepsilon > -r_L]$. As mentioned, it reflects the limited liability of shareholders and the absence of risk pricing in deposits due to deposit insurance. As $\mathbb{E}[\varepsilon|\varepsilon > -r_L] \geq \mathbb{E}\varepsilon = 0$, $R^{L,d} \geq R^L$. Using this notation, the first-order conditions can be summarized as follows:

$$R^B + \hat{\Lambda} + \hat{\Phi}_w = R^D + g_D + \lambda\hat{\Lambda} + \bar{w}\hat{\Phi}_w = R^{L,d} - g_L - \gamma\hat{\chi} = \hat{R}^E - \hat{\chi} \tag{60}$$

Showing that $\lambda \leq \bar{w} \leq 1$ Recall that $\bar{w} \equiv B/D$. $\bar{w} \geq \lambda$ follows from the liquidity requirement. Proof of $\bar{w} \leq 1$ is by contradiction. Suppose $\bar{w} > 1$. Then, from the assumption $F_w(1) = 1$, $F_w'(\bar{w}) = 0$, which would imply $\hat{\Phi}_w = 0$, using FOC($\bar{w}$). Moreover, $\bar{w} > 1$ would imply $\bar{w} > \lambda$ and thus $\hat{\Lambda} = 0$. Hence, from (60), we would have $R^B = \hat{R}^E - \hat{\chi} = (R^E - \chi)/\Phi_L$. Recall that $\chi \geq 0$. (i) If $\chi > 0$, then $E = \gamma L$, so $B = E + D - L = D - (1-\gamma)L < D$, so $\bar{w} < 1$, contradicting $\bar{w} > 1$. (ii) If $\chi = 0$, then $R^B = R^E/\Phi_L \geq R^E$. This is inconsistent with equilibrium, as the household's optimality condition (3) and the assumption that $u_b(c,d,b) > 0$ imply that $R^B < R^E$. Hence, $\bar{w} \leq 1$.

Ranking of returns From (60), as Kuhn-Tucker multipliers must be nonnegative, a finite solution requires the following ranking of returns:

$$R^B + \hat{\Phi}_w \leq R^D + g_D + \bar{w}\hat{\Phi}_w \leq R^{L,d} - g_L \leq \hat{R}^E \tag{61}$$

where the strictness of each inequality is determined only by the bindingness of the two requirements. From FOC ($\bar{w}$),

$$\hat{\Phi}_w = F'_w(\bar{w})\hat{H}(r_L)L/D \geq 0 \tag{62}$$

and since $0 \leq \bar{w} \leq 1$, the following ranking also holds

$$R^B \leq R^D + g_D \leq R^{L,d} - g_L \leq \hat{R}^E$$

Finally, as $R^L \leq R^{L,d}$,

$$R^L - g_L \leq \hat{R}^E$$

Bindingness of regulatory constraints Next, from the first equality in (60),

$$\Lambda = \frac{\Phi_L}{1-\lambda}\left(R^D + g_D - R^B - (1-\bar{w})\hat{\Phi}_w\right) \tag{63}$$

Hence, the *liquidity requirement* binds if and only if the first inequality in (61) is strict, i.e. $R^D + g_D > R^B + (1-\bar{w})\hat{\Phi}_w$. In addition, from the last equality in (60)

$$\chi = \frac{\Phi_L}{1-\gamma}\left(\hat{R}^E - (R^{L,d} - g_L)\right)$$

Hence, the *capital requirement* binds if and only if the last inequality in (61) is strict, i.e. $\hat{R}^E > R^{L,d} - g_L$.

Marginal zero profit condition Taking the weighted average of the second and fourth expressions in (60), using weights $(1-\gamma)$ and γ, respectively, and exploiting their equality with the third expression yields after some rearranging:

$$R^{L,d} - g_L = \gamma\hat{R}^E + (1-\gamma)\{R^D + g_D + \lambda\hat{\Lambda} + \bar{w}\hat{\Phi}_w\}$$

If $\Lambda = 0$, then $\hat{\Lambda} = 0$ and (63) simplifies to $\hat{\Phi}_w = (R^D - R^B + g_D)/(1-\bar{w})$, so that

$$\begin{aligned} R^{L,d} - g_L &= \gamma\hat{R}^E + (1-\gamma)\{R^D + g_D + \frac{\bar{w}}{1-\bar{w}}(R^D - R^B + g_D)\} \\ &= \gamma\hat{R}^E + (1-\gamma)\{\tilde{R}^D(\bar{w}) + \frac{1}{1-\bar{w}}g_D\} \end{aligned}$$

as $\tilde{R}^D(\bar{w}) \equiv R^D + \frac{\bar{w}}{1-\bar{w}}(R^D - R^B)$.

If $\Lambda > 0$, then $\bar{w} = \lambda$. In that case, (63) can be rewritten as $\hat{\Lambda} = \frac{1}{1-\lambda}(R^D + g_D - R^B) -$

$\hat{\Phi}_w$. Using this,

$$\begin{aligned}
R^{L,d} - g_L &= \gamma \hat{R}^E + (1-\gamma)\{R^D + g_D + \lambda(\hat{\Lambda} + \hat{\Phi}_w)\} \\
&= \gamma \hat{R}^E + (1-\gamma)\{R^D + g_D + \frac{\lambda}{1-\lambda}(R^D + g_D - R^B)\} \\
&= \gamma \hat{R}^E + (1-\gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D\}
\end{aligned}$$

Since $\bar{w} = \lambda$ when $\Lambda > 0$, we can summarize the results with the following zero profit condition that holds whether or not the liquidity requirement binds:

$$R^{L,d} - g_L = \gamma \hat{R}^E + (1-\gamma)\{\tilde{R}^D(\bar{w}) + \frac{1}{1-\bar{w}} g_D\}$$

Alternative condition for a binding capital requirement This implies that $R^{L,d} - g_L - \hat{R}^E = (1-\gamma)\{\tilde{R}^D(\bar{w}) + \frac{1}{1-\bar{w}} g_D - \hat{R}^E\}$, yielding the following equivalent, alternative condition for a binding (slack) capital requirement:

$$R^{L,d} - g_L < (=) \hat{R}^E \Leftrightarrow \tilde{R}^D(\bar{w}) + \frac{1}{1-\bar{w}} g_D < (=) \hat{R}^E$$

Zero profits Zero profits follow from

$$\begin{aligned}
\hat{R}^E \pi^B &= \hat{R}^E F_w(\bar{w}) H(r_L) L / R^E - \hat{R}^E E \\
&= (H(r_L)/H'(r_L))L - \hat{R}^E E \\
&= (r_L + \mathbb{E}[\varepsilon | \varepsilon > -r_L])L - \hat{R}^E E \\
&= R^{L,d} L + R^B(E + D - L) - R^D D - g(D, L) - \hat{R}^E E \\
&= [R^{L,d} - g_L(D,L) - R^B]L - [R^D + g_D(D,L) - R^B]D - [\hat{R}^E - R^B]E \\
&= [\hat{\Lambda} + \hat{\Phi}_w + \gamma\hat{\chi}]L - [(1-\lambda)\hat{\Lambda} + (1-\bar{w})\hat{\Phi}_w]D - [\hat{\Lambda} + \hat{\Phi}_w + \hat{\chi}]E \\
&= -\hat{\Lambda}(E + (1-\lambda)D - L) - \hat{\chi}(E - \gamma L) - \hat{\Phi}_w(E + (1-\bar{w})D - L) \\
&= 0
\end{aligned}$$

where the steps follow from (57); the definition of $\hat{R}^E$ and (58); (56); (25) and (26); Euler's theorem; the first-order conditions in (60); and the complementary slackness conditions and definition of $\bar{w}$, in that order. Since $\hat{R}^E > 0$, $\pi^B = 0$.

Zero profits imply that shareholders get R^E in expectation. Their gross return is zero with probability p_F (failure) and $\hat{R}^E = R^E/(1-p_F)$ with probability $1-p_F$ (non-failure).

Choice of $\bar{w}$ When the liquidity requirement binds, $\bar{w} = \lambda$. Otherwise, it follows from inserting $\Lambda = 0$ into (63):

$$(1 - \bar{w})\hat{\Phi}_w = R^D + g_D - R^B$$

From (62),

$$\hat{\Phi}_w = F'_w(\bar{w})\hat{H}(r_L)\frac{L}{D} = \frac{F'_w(\bar{w})H(r_L)L}{\Phi_L D}$$

Since $0 = \pi^B = F_w(\bar{w})\, H(r_L)L/R^E - E$ (see (57)), it follows that $H(r_L)L = ER^E/F_w(\bar{w})$ and so

$$\hat{\Phi}_w = \frac{F'_w(\bar{w})\hat{R}^E E}{F_w(\bar{w})\, D} \equiv z$$

Hence, when the liquidity requirement is slack, the choice of $\bar{w}$ is determined by the optimality condition

$$(1 - \bar{w})z = R^D + g_D - R^B \tag{64}$$

This expression equates the marginal benefits and costs of adding of government bonds, financed with deposits (which is always permitted by both the capital and the liquidity requirement). To see this more explicitly, rewrite (64) as

$$(1 - F_\varepsilon(-r_L))F'_w(\bar{w})(1 - \bar{w})\hat{R}^E E/D = (1 - F_\varepsilon(-r_L))F_w(\bar{w})\,(R^D + g_D - R^B)$$

Investing marginally more in bonds and financing this with deposits raises $\bar{w}$ by $(1-\bar{w})/D$,[70] which reduces the probability of failure from liquidity stress by $F'_w(\bar{w})(1-\bar{w})/D$. Conditional on no failure, shareholders expect to get $\hat{R}^E E$. Thus, the left hand-side is the expected marginal benefit to shareholders from this strategy. On the other side, investing marginally more in bonds and financing this with deposits reduces expected net cash-flow conditional on no failure by $R^D + g_D - R^B$. Thus, the right-hand side is the expected marginal cost to shareholders from this strategy.

This concludes the proof of proposition 4.

Failure probability (proof of proposition 5)

The proposition assumes that both regulatory requirements bind, so in particular, $\bar{w} = \lambda$. Inserting this into the probability of failure (59) yields:

$$p_F = 1 - F_w(\lambda)(1 - F_\varepsilon(-r_L)) \tag{65}$$

[70] Let $j(x) \equiv (B + x)/(D + x)$. Then $j'(0) = (1 - \bar{w})/D$.

Using (57), the result that $\pi^B = 0$, and the assumption of binding requirements ($\bar{w} = \lambda$, and $E/L = \gamma$), we obtain

$$F_w(\lambda)H(r_L) = \gamma R^E \tag{66}$$

The proposition also holds fixed R^E. (As noted, this can be viewed as a partial equilibrium or steady state perspective, as in steady state, $R^E = \beta^{-1}$, which is invariant to changes in the requirements.) Totally differentiate (66) with $dR^E = 0$:

$$F'_w(\lambda)H(r_L)d\lambda + F_w(\lambda)H'(r_L)dr_L = R^E d\gamma$$

For the effect of the capital requirement, set $d\lambda = 0$. Then

$$F_w(\lambda)H'(r_L)\frac{dr_L}{d\gamma} = R^E$$

Since $F_w(\lambda)$, $H'(r_L)$, and R^E are all strictly positive,[71] it follows that

$$dr_L/d\gamma > 0$$

Thus, an increase in γ raises r_L, which reduces the failure probability from credit risk, $F_\varepsilon(-r_L)$. Since λ is fixed here, the probability of failure from liquidity risk, $1 - F_w(\lambda)$, is not affected by γ. Therefore, a rise in γ reduces the overall failure probability (see (65)):

$$dp_F/d\gamma < 0$$

For the effect of the liquidity requirement, set $d\gamma = 0$. Then

$$\frac{dr_L}{d\lambda} = -\frac{F'_w(\lambda)H(r_L)}{F_w(\lambda)H'(r_L)} \leq 0$$

Thus, an increase in λ (weakly) reduces r_L and thus increases $F_\varepsilon(-r_L)$, the probability of failure from credit risk. At the same time, an increase in λ raises $F_w(\lambda)$, reducing the probability of failure from liquidity risk $(1 - F_w(\lambda))$. As a result, the combined effect on the probability of failure is theoretically ambiguous.[72] This concludes the proof of prop. 5.

[71] Strict positivity of $F_w(\lambda)$ follows from the assumption $F_w(0) > 0$, $\lambda \geq 0$, and the fact that distribution functions are increasing. Strict positivity of $H'(r_L)$ follows from $H'(r_L) = 1 - F_\varepsilon(-r_L)$ (see (55)) and the previously noted fact that $F_\varepsilon(-r_L) = 1$ would mean that the bank would fail with probability one, which is inconsistent with the supply of equity by shareholders.

[72] Note that the overall probability of failure p_F equals the sum of the probability of failure due to liquidity risk $(1 - F_w(\lambda))$ and the probability of failure due to credit risk $(F_\varepsilon(-r_L))$, minus the product of these two probabilities, reflecting the independence of the two risks. To asses the net impact on the overall failure

C.3. Competitive Equilibrium

Definition Given a government policy λ, γ, and T, an equilibrium is defined as a path of consumption, capital, employment, and financial quantities and returns, for $t = 0, 1, 2, \ldots$, such that households, banks and firms all solve their maximization problems, taxes are set according to (33), and all markets clear. In the expanded model, goods market clearing is given by:

$$F(K_t, 1) + (1-\delta)K_t = c_t + K_{t+1} + g(D_t, L_t) + \varphi L_t^{NB} + \Psi_t \tag{67}$$

Recall that Ψ_t represents total deadweight resolution costs; see (32). For all other markets, clearing conditions remain the same as in the baseline; see (12).

Dynamic system To characterize the competitive equilibrium, first use (24), (26), and (29) to derive the equity-loan spread:

$$R^E - R^L = \rho(\bar{w})\left\{(R^E - R^D) - \bar{w}(R^E - R^B) - g_D(D, L)\right\} - g_L + \mathbb{E}\left[\varepsilon | \varepsilon > -r_L\right] - \gamma(\hat{R}^E - R^E)$$

where the function ρ is defined as follows:

$$\rho(w) \equiv (1-\gamma)/(1-w)$$

Using this, the market clearing conditions (12) and (67), and equations (1) through (3) and (8) through (10) (baseline results that remain valid in the expanded model) and (23) through (26), the bank's balance sheet identity, proposition 4, and (32), the equilibrium allocation can be characterized in terms of a dynamic system in (K_t, c_t) with d_t, b_t, L_t, Ψ_t, R_t^E, $\bar{w}_t$, $r_{L,t}$, and $\hat{R}_t^E$ as auxiliary variables:

$$K_{t+1} = F(K_t, 1) + (1-\delta)K_t - c_t - g(d_t, L_t) - \varphi(K_t - L_t) - \Psi_t \tag{68}$$

$$R_t^E = \left(\beta u_c(c_t, d_t, b_t)/u_c(c_{t-1}, d_{t-1}, b_{t-1})\right)^{-1} \tag{69}$$

$$F_K(K_t, 1) + 1 - \delta = R_t^L = R_t^E - \Delta_{L,t} \tag{70}$$

---

probability, note that with both requirements binding

$$(1 - p_F)(r_L + \mathbb{E}\left[\varepsilon | \varepsilon > -r_L\right]) = R^E \gamma$$

With γ and R^E fixed and with $\frac{dr_L}{d\lambda} < 0$, the failure rate p_F is decreasing (increasing) in λ if $(r_L + \mathbb{E}\left[\varepsilon | \varepsilon > -r_L\right])$ is increasing (decreasing) in r_L. Absent further restrictions on the distribution F_ε both cases are theoretically possible, at least locally.

with the spread $\Delta_L = R^E - R^L$ equal to to:

$$\Delta_{L,t} \equiv \rho(\bar{w}_t)\left(\frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - g_D(d_t,L_t) - \bar{w}_t\frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)}\right) - g_L(d_t,L_t) + \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right] - \gamma(\hat{R}^E_t - R^E_t) \tag{71}$$

and where L_t, d_t, b_t, Ψ_t, $\bar{w}_t$, $r_{L,t}$ and $\hat{R}^E_t$ are jointly determined by the following equations:[73]

$$\text{If for } L_t = K_t,\ \Delta_{L,t} > -\varphi, \text{ then } L_t = K_t;\ \text{ else } \Delta_{L,t} = -\varphi \tag{72}$$

$$\text{If } \Delta_{L,t} > \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right] - g_L(d_t,L_t) - (\hat{R}^E_t - R^E_t), \text{ then } d_t = (1-\gamma)L_t + \bar{B} - b_t; \tag{73}$$
$$\text{else } \Delta_{L,t} = \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right] - g_L(d_t,L_t) - (\hat{R}^E_t - R^E_t) \text{ and } (d_t - \bar{B} + b_t)/(1-\gamma) \le L_t \le K_t$$

$$\Psi_t \equiv (1 - F_w(\bar{w}_t))\psi_{Liq}L_t + F_w(\bar{w}_t)F_\varepsilon(-r_{L,t})\psi_{Sol}L_t \tag{74}$$

$$\bar{w}_t = (\bar{B} - b_t)/d_t \tag{75}$$

$$(r_{L,t} + \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right])L_t = \hat{R}^E_t(L_t + \bar{B} - b_t - d_t) \tag{76}$$

$$\hat{R}^E_t = R^E_t/\{F_w(\bar{w}_t)(1 - F_\varepsilon(-r_{L,t}))\} \tag{77}$$

$$\text{If } \Delta_{B,t} > 0, \text{ then } \bar{B} - b_t = \lambda d_t;\ \text{ else } \Delta_{B,t} = 0 \tag{78}$$

with $\Delta_B = R^D + g_D - R^B - (1-\bar{w})z$ given by:

$$\Delta_{B,t} \equiv \frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - \frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} + g_D(d_t,L_t) - (1-\bar{w}_t)\frac{F'_w(\bar{w}_t)\hat{R}^E_t(L_t + \bar{B} - b_t - d_t)}{F_w(\bar{w}_t)d_t} \tag{79}$$

C.4. Planner's Problem for the Expanded Model

Equivalence with the Competitive Equilibrium

After substituting in the bond market clearing and bank balance sheet and suppressing the argument θ where this does not lead to confusion, the Lagrangian to the constrained

[73] For (73), $R^{L,d} - g_L(D,L) \le \hat{R}^E \Leftrightarrow R^L + \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right] - g_L(D,L) \le \hat{R}^E \Leftrightarrow \mathbb{E}\left[\varepsilon|\varepsilon > -r_{L,t}\right] - g_L(D,L) \le \Delta_L + \hat{R}^E - R^E$

planner's problem in (34) is:

$$\mathcal{L} = \max_{\{c_t,d_t,b_t,L_t,K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{u(c_t,d_t,b_t) + \omega_t^{sp}[F(K_t,1) + (1-\delta)K_t - c_t - K_{t+1} - g(d_t,L_t)$$
$$-\varphi(K_t - L_t) + \tau_t^L L_t - \tau_t^E(L_t + \bar{B} - b_t - d_t) + Q_t]$$
$$+\Lambda_t^{sp}[\bar{B} - b_t - (\lambda + \tau_t^{\bar{w}})d_t] + \chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t]$$
$$+\mu_t^{sp}[K_t - L_t]\}$$

resulting in first-order conditions:

$$\begin{aligned}
(c) \qquad & u_c(c_t,d_t,b_t) = \omega_t^{sp} \\
(d) \qquad & u_d(c_t,d_t,b_t) = \omega_t^{sp}(g_D(d_t,L_t) - \tau_t^E) + \Lambda_t^{sp}(\lambda + \tau_t^{\bar{w}}) + \chi_t^{sp} \\
(b) \qquad & u_b(c_t,d_t,b_t) = -\omega_t^{sp}\tau_t^E + \Lambda_t^{sp} + \chi_t^{sp} \\
(L) \qquad & \omega_t^{sp}[\varphi - g_L(d_t,L_t) + \tau_t^L - \tau_t^E] + \chi_t^{sp}(1-\gamma) = \mu_t^{sp} \\
(K) \qquad & \omega_t^{sp}[F_K(K_t,1) + 1 - \delta - \varphi] = \beta^{-1}\omega_{t-1}^{sp} - \mu_t^{sp}
\end{aligned} \tag{80}$$

with $\chi_t^{sp} \geq 0$, $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.[74]

Noting that $\lambda + \tau_t^{\bar{w}} = \bar{w}_t^{ce}$, subtract $\bar{w}_t^{ce}$ times the first-order condition with respect to bonds (FOC (b)) from FOC (d) to obtain $u_d - \bar{w}_t^{ce}u_b = (1-\bar{w}_t^{ce})\chi_t^{sp} + \omega_t^{sp}[g_D - (1-\bar{w}_t^{ce})\tau_t^E]$ (omitting arguments for brevity). Solving for χ_t^{sp}, inserting the result into FOC (L), and using FOC (c) yields:

$$\mu_t^{sp} = \omega_t^{sp}(\Delta_{L,t}^{sp} + \varphi) \tag{81}$$

with $\Delta_{L,t}^{sp}$ defined as

$$\Delta_{L,t}^{sp} \equiv \rho(\bar{w}_t^{ce})\left(\frac{u_d(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - \bar{w}_t^{ce}\frac{u_b(c_t,d_t,b_t)}{u_c(c_t,d_t,b_t)} - g_D(d_t,L_t)\right) - g_L(d_t,L_t) + \tau_t^L - \gamma\tau_t^E \tag{82}$$

(Recall that $\rho(w) = (1-\gamma)/(1-w)$.) Inserting this into FOC (K) and using FOC (c) yields:

$$F_K(K_t,1) + 1 - \delta = \beta^{-1}\frac{u_c(c_{t-1},d_{t-1},b_{t-1})}{u_c(c_t,d_t,b_t)} - \Delta_{L,t}^{sp} \tag{83}$$

Since $\tau_t^L \equiv \mathbb{E}\left[\varepsilon | \varepsilon > -r_{L,t}^{ce}\right]$ and $\tau_t^E \equiv \hat{R}_t^{E,ce} - R_t^{E,ce}$, equations (82) and (83) replicate equations (69), (70), and (71) in the characterization of the decentralized equilibrium. Fur-

[74] The social planner's liquidity requirement is an equality constraint. This specification is compatible with replication of the competitive equilibrium and simplifies some of the analysis that follows.

thermore, (72) follows from (81), $\omega_t^{sp} = u_{c,t} > 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.

Since $\chi_t^{sp}(1-\gamma) = \mu_t^{sp} + \omega_t^{sp}[-\varphi + g_L(d_t, L_t) - \tau_t^L + \tau_t^E]$ (from FOC(L)) and $\mu_t^{sp} = \omega_t^{sp}(\Delta_{L,t}^{sp} + \varphi)$ (from (81)), $\chi_t^{sp}(1-\gamma) = \omega_t^{sp}(\Delta_{L,t}^{sp} + g_L(d_t, L_t) - \tau_t^L + \tau_t^E)$. As $\omega_t^{sp} > 0$, $\chi_t^{sp} \geq 0$ and $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, it follows that $\chi_t^{sp} > 0$ and $d_t = (1-\gamma)L_t + \bar{B} - b_t$ if $\Delta_{L,t}^{sp} + g_L(d_t, L_t) - \tau_t^L + \tau_t^E > 0$; otherwise $\chi_t^{sp} = 0$, $d_t \leq (1-\gamma)L_t + \bar{B} - b_t$, and $\Delta_{L,t}^{sp} + g_L(d_t, L_t) - \tau_t^L + \tau_t^E = 0$. Since again, $\tau_t^L \equiv \mathbb{E}\left[\varepsilon | \varepsilon > -r_{L,t}^{ce}\right]$ and $\tau_t^E \equiv \hat{R}_t^{E,ce} - R_t^{E,ce}$, this result replicates (73) in the decentralized equilibrium.

Equation (68) in the characterization of the decentralized equilibrium is included as one of the constraints of the planner's problem, since from (35), $\tau_t^L L_t - \tau_t^E(L_t + \bar{B} - b_t - d_t) + Q_t = -\Psi_t^{ce}$ under replication, which will be verified to hold. Likewise, the CE's resolution costs in equation (74) are included in Q_t for the planner. Equations (76) and (77) determine the CE's auxiliary variables $r_{L,t}^{ce}$, and $\hat{R}_t^{E,ce}$. The values of these variables are hard-coded into the planner's problem through τ_t^L and τ_t^E, respectively (see (35)).

That leaves equations (75), (78), and (79) from the CE. The equations jointly determine $\bar{w}_t^{ce}$ and b_t^{ce}. The planner's liquidity requirement implies that $\bar{B} - b_t = (\lambda + \tau_t^{\bar{w}})d_t = \bar{w}_t^{ce} d_t$, thus replicating $\bar{w}_t^{ce}$ and b_t^{ce}, given that the equation for deposits (73) has already been shown to be replicated.

Collecting these results, it is apparent that the allocations of c_t, d_t, b_t, K_t, and L_t implied by the planner's problem are identical to those of the decentralized equilibrium summarized in equations (68)-(79). Hence, the constrained social planner's problem replicates the decentralized equilibrium. Moreover, under those conditions, welfare equals $V_0(\theta)$, as defined in (16).

Proof of proposition 6

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising the liquidity requirement λ is:

$$\frac{\partial V_0(\theta)}{\partial \lambda} = \sum\nolimits_{t=0}^{\infty} \beta^t \left\{ \omega_t^{sp} \left[\frac{\partial Q_t(\theta)}{\partial \lambda} + H_t(\theta) \right] - \Lambda_t^{sp} d_t \left(1 + \frac{\partial \tau_t^{\bar{w}}(\theta)}{\partial \lambda} \right) \right\}$$

where $H_t(\theta)$ collects the terms arising from the marginal effect of λ on the wedges $\tau_t^L(\theta)$ and $\tau_t^E(\theta)$ (recall that θ includes λ):

$$H_t(\theta) \equiv \frac{\partial \tau_t^L(\theta)}{\partial \lambda} L_t - \frac{\partial \tau_t^E(\theta)}{\partial \lambda} E_t$$

Here $E_t = L_t + \bar{B} - b_t - d_t$. Recalling that $Q_t \equiv \tau_t^E E_t^{ce} - \tau_t^L L_t^{ce} - \Psi_t^{ce}$ (omitting the argument θ) and that all CE values may depend on λ,

$$\frac{\partial Q_t}{\partial \lambda} = -H_t^{ce} + \tau_t^E \frac{\partial E_t^{ce}}{\partial \lambda} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \lambda} - \frac{\partial \Psi_t^{ce}}{\partial \lambda}$$

where H_t^{ce} is the CE version of H_t. Since we have shown that the planner's allocation is identical to the CE's allocation (in particular, $L_t = L_t^{ce}$ and $E_t = E_t^{ce}$), $H_t = H_t^{ce}$. Hence,

$$\frac{\partial Q_t}{\partial \lambda} + H_t = \tau_t^E \frac{\partial E_t^{ce}}{\partial \lambda} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \lambda} - \frac{\partial \Psi_t^{ce}}{\partial \lambda}$$

Further, recall that $\tau_t^{\bar{w}}(\theta) \equiv \bar{w}_t^{ce} - \lambda$, so that

$$1 + \frac{\partial \tau_t^{\bar{w}}}{\partial \lambda} = \frac{\partial \bar{w}_t^{ce}}{\partial \lambda} = \begin{Bmatrix} 1 \text{ if } \bar{w}_t^{ce} = \lambda \\ 0 \text{ if } \bar{w}_t^{ce} > \lambda \end{Bmatrix} = 1_{\{\bar{w}_t^{ce} = \lambda\}}$$

which is an indicator variable for a binding liquidity requirement in CE. Therefore,

$$\frac{\partial V_0(\theta)}{\partial \lambda} = \sum\nolimits_{t=0}^{\infty} \beta^t \left\{ \omega_t^{sp} \left[\tau_t^E \frac{\partial E_t^{ce}}{\partial \lambda} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \lambda} - \frac{\partial \Psi_t^{ce}}{\partial \lambda} \right] - \Lambda_t^{sp} d_t 1_{\{\bar{w}_t^{ce} = \lambda\}} \right\}$$

Next, we derive an expression for Λ_t^{sp}. Taking the difference between the planner's FOC(b) and FOC(d) (see (80)) yields after cancelling terms and using FOC(c):

$$\Lambda_t^{sp} = \{u_b(c_t, d_t, b_t) - u_d(c_t, d_t, b_t) + u_c(c_t, d_t, b_t) g_D(d_t, L_t)\} / (1 - (\lambda + \tau_t^{\bar{w}})) \qquad (84)$$

Λ_t^{sp} is the planner's shadow cost of the liquidity requirement. Banks satisfy the liquidity requirement by holding bonds financed with deposits. This activity reduces bonds available to households, but increases their deposits. At the margin, the net reduction in the utility of liquidity services available to households is $u_b - u_d$. Deposit servicing also entails noninterest costs, whence the expression in (84) for the cost of the liquidity requirement.

As mentioned, terms reflecting reduced liquidity are classified as welfare *costs*, while terms related to reductions in resolution costs and distortions are classified as welfare *benefits*. Accordingly, the marginal welfare costs in utility units ($\nu_{Liq}^{Cost,u}$) are:

$$\begin{aligned} \nu_{Liq}^{Cost,u} &= \sum\nolimits_{t=0}^{\infty} \beta^t \Lambda_t^{sp} d_t 1_{\{\bar{w}_t^{ce} = \lambda\}} \\ &= \sum\nolimits_{t=0}^{\infty} \beta^t \{u_b(c_t, d_t, b_t) - u_d(c_t, d_t, b_t) + u_c(c_t, d_t, b_t) g_D(d_t, L_t)\} \frac{1_{\{\bar{w}_t^{ce} = \lambda\}}}{1 - \lambda} d_t \end{aligned}$$

where the last step uses (84) and the fact that $\tau_t^{\bar{w}} = 0$ when $\bar{w}_t^{ce} = \lambda$. In addition, the marginal welfare benefits in utility units ($\nu_{Liq}^{Ben,u}$) equal:

$$\nu_{Liq}^{Ben,u} = \sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t) \left\{ \tau_t^E \frac{\partial E_t^{ce}}{\partial \lambda} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \lambda} - \frac{\partial \Psi_t^{ce}}{\partial \lambda} \right\}$$

The marginal welfare cost can be expressed in terms of observables. Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, then their equilibrium values can be used. Moreover, in that equilibrium, we have, by taking the difference between the household's first-order conditions,

$$u_b(c_t, d_t, b_t) - u_d(c_t, d_t, b_t) = u_c(c_t, d_t, b_t)(R_t^D - R_t^B) \tag{85}$$

Hence,

$$\nu_{Liq}^{Cost,u} = \sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t) \left\{ R_t^D - R_t^B + g_D(d_t, L_t) \right\} \frac{1_{\{\bar{w}_t = \lambda\}}}{1 - \lambda} d_t$$

It is standard to compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$, which equals, to a first-order approximation, $\sum_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t) c_t \nu$ in utility units. Equating this to the right-hand side of the previous equation and solving for ν yields for the marginal welfare cost in relative consumption equivalents (ν_{Liq}^{Cost}):

$$\nu_{Liq}^{Cost} = \sum\nolimits_{t=0}^{\infty} \varpi_t \frac{d_t}{c_t} \left\{ R_t^D - R_t^B + g_D(d_t, L_t) \right\} \frac{1_{\{\bar{w}_t = \lambda\}}}{1 - \lambda} \tag{86}$$

where the weights $\varpi_t \equiv \frac{\beta^t u_c(c_t, d_t, b_t) c_t}{\sum_{s=0}^{\infty} \beta^s u_c(c_s, d_s, b_s) c_s}$ are the same weights as in (51), a roughly geometrically declining series of positive weights that sum to one. Similarly, the marginal welfare benefit expressed in relative consumption equivalents (ν_{Liq}^{Ben}) is:

$$\nu_{Liq}^{Ben} = \sum\nolimits_{t=0}^{\infty} \frac{\varpi_t}{c_t} \left\{ \tau_t^E \frac{\partial E_t}{\partial \lambda} - \tau_t^L \frac{\partial L_t}{\partial \lambda} - \frac{\partial \Psi_t}{\partial \lambda} \right\} \tag{87}$$

where we have dropped the *ce* superscript, as the planner and CE values are identical for any λ, which implies that their derivatives with respect to λ are also equal.

The expression for the cost can be simplified further with the assumption that the economy is in steady state in the current period 0. In that case, all variables are constant over time, other than the weights, which sum to one. Hence,

$$\nu_{Liq}^{Cost} = \frac{d_0}{c_0}(R_0^D - R_0^B + g_D(d_0, L_0)) \frac{1_{\{\bar{w}_0 = \lambda\}}}{1 - \lambda}$$

The proposition follows from $1_{\{\bar{w}_0 = \lambda\}} = 1$ when the liquidity requirement binds, so that then

ν_{Liq}^{Cost} equals the baseline model's ν_{Liq} in (17).[75] Outside a current-period steady state, (86) applies, and with a binding liquidity requirement ($\bar{w}_t = \lambda$), this is identical to (50), the analogous result for the baseline model. ν_{Liq}^{Cost} is the marginal welfare cost of the liquidity requirement expressed as the welfare-equivalent permanent relative loss in consumption. Thus, a first-order approximation of the gross welfare cost of permanently increasing the liquidity requirement λ by $\Delta\lambda$ is $\nu_{Liq}^{Cost}\Delta\lambda$.

Proof of proposition 7

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising the capital requirement γ is:

$$\frac{dV_0(\theta)}{d\gamma} = \sum\nolimits_{t=0}^{\infty} \beta^t \left\{ \omega_t^{sp} \left[\frac{\partial Q_t(\theta)}{\partial \gamma} + J_t(\theta) \right] - \Lambda_t^{sp} d_t \frac{\partial \tau_t^{\bar{w}}(\theta)}{\partial \gamma} - \chi_t^{sp} L_t \right\}$$

where $J_t(\theta)$ collects the terms arising from the marginal effect of γ on the wedges $\tau_t^L(\theta)$ and $\tau_t^E(\theta)$ (recall that θ includes γ):

$$J_t \equiv \frac{\partial \tau_t^L(\theta)}{\partial \gamma} L_t - \frac{\partial \tau_t^E(\theta)}{\partial \gamma} E_t$$

where $E_t = L_t + \bar{B} - b_t - d_t$. Recalling that $Q_t \equiv \tau_t^E E_t^{ce} - \tau_t^L L_t^{ce} - \Psi_t^{ce}$ (omitting the argument θ) and that all CE values may depend on γ,

$$\frac{\partial Q_t}{\partial \gamma} = -J_t^{ce} + \tau_t^E \frac{\partial E_t^{ce}}{\partial \gamma} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \gamma} - \frac{\partial \Psi_t^{ce}}{\partial \gamma}$$

where J_t^{ce} is the CE version of J_t. Since we have shown that the planner's allocation is identical to the CE's allocation (in particular, $L_t = L_t^{ce}$ and $E_t = E_t^{ce}$), $J_t = J_t^{ce}$. Hence,

$$\frac{\partial Q_t}{\partial \gamma} + J_t = \tau_t^E \frac{\partial E_t^{ce}}{\partial \gamma} - \tau_t^L \frac{\partial L_t^{ce}}{\partial \gamma} - \frac{\partial \Psi_t^{ce}}{\partial \gamma}$$

[75] Unfortunately, the benefits appear to barely simplify from imposing a steady state, as we see no guarantee that terms such as $\frac{\partial E_t}{\partial \lambda}$ would be constant over t (even though E_t is) due to the transition to a new steady state associated with a change in λ. Thus, with period 0 in steady state, we merely have:

$$\nu_{Liq}^{Ben} = \frac{1-\beta}{c_0} \left(\tau_0^E \left(\sum\nolimits_{t=0}^{\infty} \beta^t \frac{\partial E_t}{\partial \lambda} \right) - \tau_0^L \left(\sum\nolimits_{t=0}^{\infty} \beta^t \frac{\partial L_t}{\partial \lambda} \right) - \sum\nolimits_{t=0}^{\infty} \beta^t \frac{\partial \Psi_t}{\partial \lambda} \right)$$

Take the planner's FOC(d) and subtract λ times FOC(b) (see (80)) to obtain, after some rearranging,

$$\chi_t^{sp} = \frac{1}{1-\lambda}\{u_d(c_t,d_t,b_t) - \lambda\, u_b(c_t,d_t,b_t) - u_c(c_t,d_t,b_t)g_D(d_t,L_t) - \Lambda_t^{sp}\tau_t^{\bar{w}}\} + u_c(c_t,d_t,b_t)\tau_t^E$$

As mentioned, terms reflecting reduced liquidity are classified as welfare *costs*, while terms related to reductions in resolution costs and distortions are classified as welfare *benefits*. Accordingly, the marginal welfare costs in utility units ($\nu_{Cap}^{Cost,u}$) equal:[76]

$$\nu_{Cap}^{Cost,u} = \sum\nolimits_{t=0}^{\infty} \beta^t \frac{L_t}{1-\lambda}\left\{u_d(c_t,d_t,b_t) - \lambda\, u_b(c_t,d_t,b_t) - u_c(c_t,d_t,b_t)g_D(d_t,L_t)\right\}$$

and the marginal welfare benefits in utility units ($\nu_{Cap}^{Ben,u}$) equal:

$$\nu_{Cap}^{Ben,u} = \sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t,d_t,b_t)\left\{\tau_t^E\left(\frac{\partial E_t^{ce}}{\partial\gamma} - L_t\right) - \tau_t^L\frac{\partial L_t^{ce}}{\partial\gamma} - \frac{\partial\Psi_t^{ce}}{\partial\gamma}\right\} + \beta^t\Lambda_t^{sp}\left(\frac{\tau_t^{\bar{w}}L_t}{1-\lambda} - d_t\frac{\partial\tau_t^{\bar{w}}}{\partial\gamma}\right)$$

From equations (84) and (85), $\Lambda_t^{sp} = u_c(c_t,d_t,b_t)\left\{R_t^D - R_t^B + g_D(d_t,L_t)\right\}\frac{1}{1-\bar{w}_t}$. This factor only matters if the liquidity requirement is slack in CE, for otherwise $\tau_t^{\bar{w}} = 0$ and $\partial\tau_t^{\bar{w}}/\partial\gamma = 0$. Under a slack liquidity requirement, from proposition 4, $R_t^D - R_t^B + g_D(d_t,L_t) = (1-\bar{w}_t)z_t$, so that $\Lambda_t^{sp} = u_c(c_t,d_t,b_t)z_t$ in that case. Furthermore, when the capital requirement binds, $E_t^{ce} = \gamma L_t^{ce}$, so $\partial E_t^{ce}/\partial\gamma = L_t^{ce} + \gamma\partial L_t^{ce}/\partial\gamma$ in that case. Otherwise, $\partial E_t^{ce}/\partial\gamma = L_t^{ce}/\partial\gamma = 0$. In addition, $L_t = L_t^{ce}$. Hence, $\tau_t^E\left(\partial E_t^{ce}/\partial\gamma - L_t\right) - \tau_t^L\partial L_t^{ce}/\partial\gamma = (\gamma\tau_t^E - \tau_t^L)\partial L_t^{ce}/\partial\gamma - \tau_t^E L_t 1_{\{E_t>\gamma L_t\}}$. Combining these results,

$$\nu_{Cap}^{Ben,u} = \sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t,d_t,b_t)\left\{(\gamma\tau_t^E - \tau_t^L)\frac{\partial L_t^{ce}}{\partial\gamma} - \tau_t^E L_t 1_{\{E_t>\gamma L_t\}} - \frac{\partial\Psi_t^{ce}}{\partial\gamma} + z_t\left(\frac{\tau_t^{\bar{w}}L_t}{1-\lambda} - d_t\frac{\partial\tau_t^{\bar{w}}}{\partial\gamma}\right)\right\}$$

The marginal welfare cost can be expressed in terms of observables. Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, their equilibrium values can be used. Moreover, in that equilibrium, we have, from the household's first-order conditions (2) and (3),

$$\begin{aligned}
u_d(c_t,d_t,b_t) - \lambda u_b(c_t,d_t,b_t) &= u_c(c_t,d_t,b_t)\{R_t^E - R_t^D - \lambda(R_t^E - R_t^B)\}\\
&= u_c(c_t,d_t,b_t)\{(1-\lambda)(R_t^E - R_t^D) - \lambda(R_t^D - R_t^B)\}\\
&= u_c(c_t,d_t,b_t)(1-\lambda)(R_t^E - \tilde{R}_t^D(\lambda))
\end{aligned}$$

[76] In line with the classification principle, the terms $-u_c\tau^E$ and $\Lambda^{sp}\tau^{\bar{w}}$ in χ^{sp} are grouped with the benefits, as these reflect distortions. Note that if the liquidity requirement binds, all terms containing $\tau^{\bar{w}}$ or $\partial\tau^{\bar{w}}/\partial\gamma$ as a factor equal zero.

Hence, the marginal welfare costs equal

$$\nu_{Cap}^{Cost,u} = \sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t) \left\{ R_t^E - \tilde{R}_t^D(\lambda) - \frac{g_D(d_t, L_t)}{1-\lambda} \right\} L_t$$

Again, as is standard, we compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$. The marginal welfare cost in relative consumption equivalents (ν_{Cap}^{Cost}) is:

$$\nu_{Cap}^{Cost} = \sum\nolimits_{t=0}^{\infty} \varpi_t \frac{L_t}{c_t} \left\{ R_t^E - \tilde{R}_t^D(\lambda) - \frac{g_D(d_t, L_t)}{1-\lambda} \right\} \tag{88}$$

where the weights ϖ_t are the same as before (see (51)).

Similarly, the marginal welfare benefit in relative consumption equivalents (ν_{Cap}^{Ben}) is:

$$\nu_{Cap}^{Ben} = \sum\nolimits_{t=0}^{\infty} \frac{\varpi_t}{c_t} \left\{ (\gamma \tau_t^E - \tau_t^L) \frac{\partial L_t}{\partial \gamma} - \tau_t^E L_t 1_{\{E_t > \gamma L_t\}} - \frac{\partial \Psi_t}{\partial \gamma} + z_t \left(\frac{\tau_t^{\bar{w}} L_t}{1-\lambda} - d_t \frac{\partial \tau_t^{\bar{w}}}{\partial \gamma} \right) \right\}$$

where we have dropped the *ce* superscript, as the planner and CE values are identical for any γ, which implies that their derivatives with respect to γ are also equal. Under a binding capital requirement this simplifies to (37) in proposition 7.

The expression for the cost can be simplified further with the assumption that the economy is in steady state in the current period 0. In that case, all variables are constant over time, other than the weights, which sum to one. Hence,

$$\nu_{Cap}^{Cost} = \frac{L_0}{c_0} \left\{ R_0^E - \tilde{R}_0^D(\lambda) - \frac{g_D(d_0, L_0)}{1-\lambda} \right\}$$

the same expression as (18) in proposition 3. Outside a current-period steady state, (88) applies, which is identical to (52), the analogous result for the baseline model. ν_{Cap}^{Cost} is the marginal welfare cost of the capital requirement expressed as the welfare-equivalent permanent relative loss in consumption. Thus, a first-order approximation of the gross welfare cost of permanently increasing the liquidity requirement γ by $\Delta\gamma$ is $\nu_{Cap}^{Cost} \Delta\gamma$.

Appendix D. Accuracy of the first-order linear approximation of the welfare costs

This appendix evaluates the quality of first-order approximations of the welfare costs of discrete changes to the capital and liquidity requirements, such as those presented in section 5 of the paper. In section 5, these first-order approximations were based on the analytical results derived in section 4, which provide exact expressions of the marginal welfare costs. Here, all results are based on an exact (up to high numerical accuracy) solution of the model, using a calibration that matches the spreads and ratios in the formulas for the marginal welfare costs, among other calibration targets. By construction, therefore, any differences between the exact and the first-order approximate welfare costs will be due to error in that approximation.

The appendix first describes the calibration of the model, then it explains the global nonlinear method for obtaining exact numbers for the welfare costs, and finally it presents detailed results. The global solution is based on a recursive representation of the economy and includes transition dynamics to compute exact welfare effects, which are compared with the first-order approximations.

Calibration

In the calibration, a subset of parameters will be chosen so that the model's steady state has exactly the same marginal welfare cost as measured in section 5 (and on which table 1 is based). Thus, as noted, any difference between the exact welfare cost of a discrete change in the regulation and the first-order approximate numbers in table 1 will be due to error in that first-order approximation. In addition, evaluating the welfare effects of very small regulatory changes using this procedure will provide a numerical check of propositions 2 and 3, which were proved analytically (of course, besides evaluating approximation error, the procedure also provides an additional numerical check of propositions 2 and 3, which were proved analytically). For the remaining parameters, and for functional forms, standard choices are made wherever possible. The details are as follows.

Starting with functional forms, the production function is assumed to be Cobb-Douglas, with capital share α_K:

$$F(K, H) = AK^{\alpha_K} L^{1-\alpha_K}.$$

The bank cost function is assumed to be linear:

$$g(d, L) = n_d d + n_L L,$$

where α_K, A, n_d and n_L are positive constants.

The derived utility function is assumed to be CRRA with separability between consumption and liquidity services. Following Poterba and Rotemberg (1985), Begenau and Landvoigt (2022) and others, it takes the following form:

$$u(c,d,b) = \frac{c^{1-\sigma_u}}{1-\sigma_u} + \theta_v \frac{((d^{(e-1)/e} + ab^{(e-1)/e})^{e/(e-1)})^{1-\sigma_v}}{1-\sigma_v},$$

with $\sigma_u > 0$, $\sigma_v > 0$, $\theta_v \geq 0$, $a \geq$ and $e > 0$.

In the benchmark calibration, we target a mixed finance steady state. Using a similar calibration strategy to Van den Heuvel (2008), the length of a period is taken to be one year and we pick standard values for the macro parameters: the capital share α_K is set at 0.3, depreciation rate δ at 0.12 and the rate of time preference β at 1/1.06. Productivity A is normalized so that steady state output is equal to 100. Following Begenau and Landvoigt (2022), risk aversion σ_u is set at 2, the curvature on the utility derived from liquidity services σ_v is set at 1.6, and the elasticity of substitution between the liquidity services from deposits and government bonds $\frac{e-1}{e}$ is set to 0.20 ($e = 1.25$). Moving on to the banking and regulatory parameters, using aggregate Tier 1 capital ratios for all banks with at least \$2B (in constant 2024 Q4 dollars) in total consolidated assets, and taking the mean of that aggregate ratio over the years in each measurement period: 1993 to 2006 for the pre-Basel III calibration and 2016 to 2019 for the Basel III calibration, the capital requirement γ is set at 0.098 and 0.130, respectively.[78] The liquidity requirement λ is set at 0 for the pre-Basel calibration and at 0.170 for the Basel III calibration. The government bond supply $\bar{B}$ is set at 42.5% of GDP, which is the average federal debt held by private investors to GDP ratio from 1986 to 2019. Lastly, the marginal cost of deposits, n_d, is set at 0.0122 (see section 5 for details).

The remaining parameters, $(n_L, a, \varphi, \theta_v)$, are chosen to obtain the same marginal welfare cost as measured in section 5. To that end the model is calibrated to match either the historically observed U.S. deposit-to-consumption ratio or the loans-to-consumption ratio, and both the equity-deposit and deposit-bonds spreads. For consumption, we use personal consumption expenditures from NIPA and for loans, we use total assets net of U.S. Treasuries and excess reserves from the HSOB. The spreads are calculated as described in section 5. Additionally, the mixed finance baseline is disciplined by matching the share of bank loans out of total loans to its value in the data, 0.42 (Durdu and Zhong (2023)).

The top block of Table D1 shows the calibrated parameters, and the bottom block

[78]We focus on Tier 1 rather than Common Equity Tier 1 because the latter is not available for the pre-Basel period.

shows the targeted model moments. The first two columns correspond to pre-Basel targets and the last two columns to Basel-III targets. Columns (1) and (3) target the deposit-to-consumption ratio to compare with the liquidity requirement's marginal welfare cost and columns (2) and (4) target the loans-to-consumption ratio to compare with the capital requirement's welfare cost. For robustness, a calibration based on a pure bank finance equilibrium is also conducted at the end of the section.

Table D1: Calibration for the Mixed Finance Steady State

		Value (different targets)				
		Pre-Basel III d_{ss}/c_{ss} (1)	Pre-Basel III L_{ss}/c_{ss} (2)	Basel III d_{ss}/c_{ss} (3)	Basel III L_{ss}/c_{ss} (4)	
Parameter						*Source*
γ	Cap. Req.	0.098	0.098	0.130	0.130	Call Reports
λ	Liq. Req.	0	0	0.170	0.170	Call Reports
α_K	Capital share	0.30				Standard
δ	Depreciation rate	0.12				Standard
n_D	Mg. cost deposits	0.0122				Measured in section 5
n_L	Mg. cost loans	0.06	0.02	0.02	0.02	Calibrated
A	TFP	22.88	21.59	21.57	21.57	Calibrated
β	Discount factor	1/1.06				Standard
σ_u	U risk aversion	2.0				Begenau and Landvoigt (2022)
σ_v	V risk aversion	1.6				Begenau and Landvoigt (2022)
e	CES liq. pref	1.25				Begenau and Landvoigt (2022)
a	CES liq. pref	0.70	0.58	0.37	0.30	Calibrated
φ	Mixed finance cost	0.040	0.001	0.001	0.001	Calibrated
θ_v	Ut. coeff. liquidity	0.02	0.02	0.02	0.02	Calibrated
$\bar{B}$	Gov. bond supply	0.425 Y^*				Measured FRED
Data-based targets and model moments						
Y^*	100.00	100.00	100.00	100.00	100.00	Normalized
d_{ss}/c_{ss}	0.66 or 0.94	0.66		0.94		FDIC and NIPA
L_{ss}/c_{ss}	0.91 or 1.07		0.91		1.07	FDIC and NIPA
$R^D - R^B$ %	-0.64 or -0.81	-0.64	-0.64	-0.81	-0.81	Measured in section 5
$R^E - R^D$ %	3.20 or 3.36	3.20	3.20	3.36	3.36	Measured in section 5
L_{ss}/K_{ss}	0.42	0.41	0.43	0.42	0.50	Durdu and Zhong (2023)

Recursive representation and global nonlinear solution

The constrained social planner's problem (where $'$ denotes next-period variables) is:

$$
\begin{aligned}
V(K) &= \max_{c,d,b,L,K'} u(c,d,b) + \beta V(K') \; s.t. \\
c + K' &= F(K,1) + (1-\delta)K - (n_d d + n_L L) - \varphi(K - L) \text{ with mult. } \omega^{sp} \\
\bar{B} - b &\geq \lambda d \text{ with mult. } \Lambda^{sp} \\
(1-\gamma)L + \bar{B} - b &\geq d \text{ with mult. } \chi^{sp} \\
K &\geq L \text{ with mult. } \mu^{sp}.
\end{aligned} \tag{89}
$$

Using this recursive representation, we implement the global nonlinear solution method from Mendoza and Villalvazo (2020) to derive equilibrium decision rules. This approach effectively handles the occasionally binding constraints arising from the liquidity requirement, the capital requirements, and the mixed financing possibility (the nonnegativity constraint on NBFI lending), while enabling us to obtain accurate welfare changes using transitional dynamics following regulatory changes. By employing this fixed-point iteration method to solve the model's equilibrium Euler equations, we obtain precise global nonlinear solutions that allow us to do welfare analysis with a high degree of confidence. For the numerical results, welfare effects are obtained simulating the economy forward and hence include any transition effects.

Results: measuring the accuracy of the first-order linear solution

Figure D1 shows the welfare cost (the negative of the change in welfare) for a range of regulatory changes. The left panel shows results based on the pre-Basel III calibration, and the right panel is for the Basel III calibration. On the horizontal axis, Δ represents the change in a requirement, either the liquidity requirement (shown in blue) or the capital requirement (shown in red). Dashed lines depict exact numerical results, while solid lines trace out the analytical first-order approximation.

Several results emerge. First, as expected, the numerical results and the first-order approximation are virtually identical for small regulatory changes (small Δ), as can be seen in the tangency of the solid and dashed lines at $\Delta = 0$. Second, more meaningful deviations for larger changes (up or down) are more visible (and suggestive of convexity). However, the differences between the solid and dashed lines do not alter the main conclusions of the paper. Specifically, consistent with the analytical results, the welfare cost of liquidity regulation is small compared to the welfare cost of the capital requirement, which is one order of magnitude larger. Table D2 shows the welfare costs after an increase of 10 percentage points ($\Delta = 0.10$) in the liquidity requirement or the capital requirement using the linear formulas of Section 4 and the global nonlinear solution. Similar to the capital requirement's welfare cost in Van den Heuvel (2008), the difference between the first-order linear approximation and the nonlinear global solution is small and below 10 percent.

Lastly, section 5.4 of the paper estimated the welfare cost of a government-imposed switch to narrow banking to be about 2.4 percent of consumption (based on the Basel III measurement period). Using the global nonlinear solution and averaging across the two Basel III period calibrations, the exact welfare cost is 3.01 percent (2.54 and 3.49 percent for the calibrations in column 3 and column 4, respectively).

Figure D1: Welfare Cost in Mixed Finance Calibration

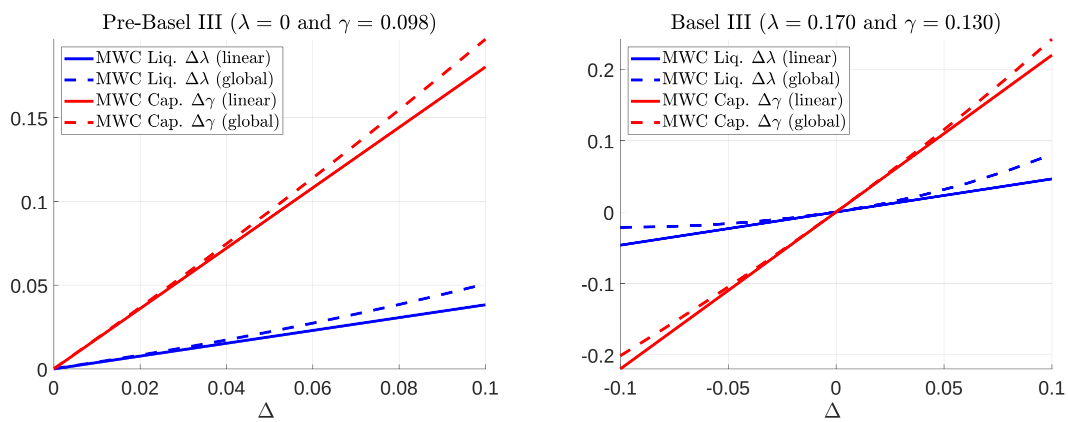


Table D2: Welfare Cost Accuracy in Mixed Finance Steady State

	Measurement period	
Welfare cost of:	Pre-Basel III	Basel III
10% Liquidity requirement		
Linear	0.038	0.046
Nonlinear	0.051	0.081
10% Capital requirement		
Linear	0.180	0.220
Nonlinear	0.197	0.242

Note: Entries correspond to the permanent consumption loss, expressed in percent, that is welfare-equivalent to a 10 percentage point increase in each requirement.

Calibration and results under a pure bank finance steady state

For robustness, a calibration based on a pure bank finance equilibrium is also conducted. In this environment, matching the historically observed deposit-consumption ratio or the loans-to-consumption ratio requires picking a nonstandard value for one of the macro parameters. Specifically, in this case, α_K is chosen to match these ratios, resulting in a lower capital share that takes values between 0.11 and 0.16. Additionally, we set the marginal cost of loans to 0.74% to match the ratio of net noninterest costs to loans. The rest of the calibration is the same as in the mixed finance baseline. The top block of Table D3 shows the calibrated parameters and the bottom block shows the targeted model moments. Tables D3, D4, and Figure D2 show the results for the pure bank finance calibration, which are similar to the baseline mixed finance steady state case.

Table D3: Calibration for the Pure Bank Finance Steady State

		Value (different targets)				
		Pre-Basel III d_{ss}/c_{ss} (1)	Pre-Basel III L_{ss}/c_{ss} (2)	Basel III d_{ss}/c_{ss} (3)	Basel III L_{ss}/c_{ss} (4)	
Parameter						*Source*
γ	Cap. Req.	0.098	0.098	0.130	0.130	Call Reports
λ	Liq. Req.	0	0	0.170	0.170	Call Reports
α_K	Capital share	0.11	0.14	0.14	0.16	Calibrated
δ	Depreciation rate	0.12				Standard
n_D	Mg. cost deposits	0.0122				Measured in section 5
n_L	Mg. cost loans	0.0074				Measured in section 5
A	TFP	62.33	54.78	55.36	48.92	Calibrated
β	Discount factor	1/1.06				Standard
σ_u	U risk aversion	2.0				Begenau and Landvoigt (2022)
σ_v	V risk aversion	1.6				Begenau and Landvoigt (2022)
e	CES liq. pref	1.25				Begenau and Landvoigt (2022)
a	CES liq. pref	0.61	0.52	0.32	0.26	Calibrated
φ	Mixed finance cost	0.0				Irrelevant for pure bank finance
θ_v	Ut. coeff. liquidity	0.02	0.02	0.01	0.01	Calibrated
$\bar{B}$	Gov. bond supply	0.425 Y^*				Measured FRED
Data-based targets and model moments						
Y^*	100.00	100.00	100.00	100.00	100.00	Normalized
d_{ss}/c_{ss}	0.66 or 0.94	0.66		0.94		FDIC and NIPA
L_{ss}/c_{ss}	0.91 or 1.07		0.91		1.07	FDIC and NIPA
$R^D - R^B$ %	-0.64 or -0.81	-0.64	-0.64	-0.81	-0.81	Measured in section 5
$R^E - R^D$ %	3.20 or 3.36	3.20	3.20	3.36	3.36	Measured in section 5

The results are summarized in Figure D2. As for the mixed finance calibration, the solid and dashed lines are tangent at $\Delta = 0$, and the numerical results and the first-order approximations are virtually identical for small regulatory changes (small Δ). For larger regulatory changes, on the order of 10 percentage points up or down, the convexity is more pronounced than in the mixed finance calibration, especially for the liquidity requirement. However, the first-order approximation still provides reasonable guidance, and the welfare cost of liquidity regulation is still small compared to the welfare cost of the capital requirement.

Figure D2: Welfare Cost in Pure Bank Finance Calibration

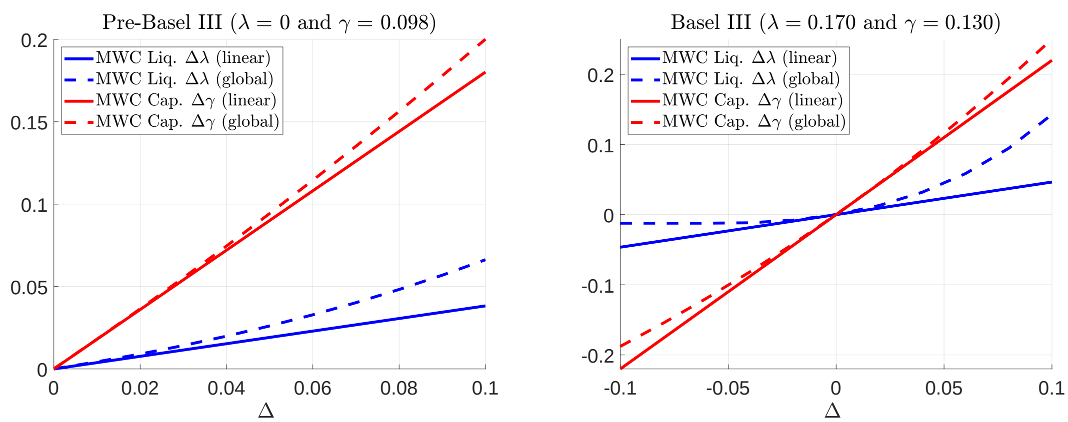


Table D4: Welfare Cost Accuracy in Pure Bank Finance Steady State

	Measurement period	
Welfare cost of:	Pre-Basel III	Basel III
10% Liquidity requirement		
Linear	0.038	0.046
Nonlinear	0.066	0.143
10% Capital requirement		
Linear	0.180	0.220
Nonlinear	0.200	0.250

Note: Entries correspond to the permanent consumption loss, expressed in percent, that is welfare-equivalent to a 10 percentage point increase in each requirement.

Appendix E. Expanded model analysis

This appendix describes the calibration, numerical solution, and detailed quantitative results for the expanded model presented in section 6, which incorporates bank liquidity and credit risk and allows banks to fail with positive probability. This expanded framework enables a quantitative analysis of welfare benefits arising from regulation. After presenting the calibration strategy and the solution method, the appendix provides a quantification of both the welfare costs and the regulatory benefits, which emerge from addressing the externalities of bank failures. Finally, it presents detailed results for optimal regulation and provides a range of estimates.

Calibration

This section details the calibration strategy employed in the expanded model presented in section 6, following closely the methodology established previously for the baseline model. Table E1 summarizes the calibrated parameters, empirical targets, and resulting model moments. Below, we elaborate on the additional parameters relative to the baseline model.

For the credit risk ϵ shocks, we employ a truncated Normal distribution with symmetric upper and lower bounds centered at zero. Both the zero mean and bounded support are assumptions maintained from the theoretical analysis. Specifically, it is assumed that $F_\epsilon \sim Truncated\ N(0, \sigma_\epsilon)$ with lower bound $\underline{\epsilon} = -F(K, H)/K$ and upper bound $\bar{\epsilon} = -\underline{\epsilon}$, where K and H refer to their steady state values. The parameter σ_ϵ is calibrated to match the empirical bank failure probability of 1.07% in the pre-Basel period documented in Corbae and D'Erasmo (2021). While our analysis focuses on the current Basel III period, we view this period as too short to reliably estimate its bank failure rate, as bank failures tend to come in waves. Since the model implies that failure rates strongly depend on regulation (which was less stringent in the pre-Basel III period), we obtain the implied bank failure rate for the pre-Basel requirements and match that to 1.07%.

For liquidity risk, we model deposit withdrawal rate shocks, w, using a Beta distribution, $F_w \sim Beta(arg1, arg2)$, which conveniently has support over $[0, 1]$. The parameters $arg1$ and $arg2$ are calibrated to match key empirical moments from banks' call reports consolidated to the top holder BHC: specifically, the average deposit outflow of 0.75% and its 99th percentile which is 8.64%. The 99th percentile is included because bank failures tend to be driven by tail events and are important to the model's welfare implications. These moments are computed from annual deposit flows (expressed as growth rates) for banks with at least \$2B (in constant 2024 Q4 dollars) in total consolidated assets from 1986 to

2019.[79] To properly estimate withdrawal shocks, we define deposits outflows as the absolute value of the negative part of deposit growth, thus replacing positive growth observations with zeros, (i.e., no net withdrawals, treated as $w = 0$), then calculate cross-sectional moments for each year, and finally take the median values of these moments across years. Our calibrated distribution also provides excellent fit across multiple moments of the empirical distribution, including the standard deviation, and the 90th and 95th percentiles.

Resolution cost parameters are calibrated to match Deposit Insurance Fund (DIF) losses relative to deposits in failed banks of 25.96%, calculated from the FDIC's Bank Failure and Assistance Data. In the model, these DIF losses are (see equations (32) and (33)):

$$DIF_t = \Psi_t - (1 - F_w(\bar{w}_t))r_{L,t} - F_w(\bar{w}_t) \int_{\underline{\epsilon}}^{-r_{L,t}} (r_{L,t} + \epsilon) L_t dF_\epsilon(\epsilon) \tag{90}$$

Absent data breaking down failures by cause (liquidity or solvency), we parsimoniously assume that $\psi_{liq} = \psi_{sol} \equiv \psi$. For robustness, a range of optimal requirements estimates for different values of deposit costs, resolution costs, and failure rates is also conducted at the end of the section.

The remaining parameters are calibrated in the same fashion as for the baseline model (see Appendix D). Specifically, as before, the calibration matches the spreads and ratios in the formulas for the marginal welfare costs (propositions 2 and 3, which were shown to remain valid for the expanded model in propositions 6 and 7). By construction, therefore, the numerical marginal welfare costs match their measurements in the main text (see section 5) up to numerical error. Additionally, as an untargeted moment, the model generates a low share of liquidity driven failures in the pre-Basel III period of 11.8%, which is consistent with Correia et al. (2026).

Solution method

We solve for the steady state of the expanded model under a mixed finance equilibrium (calibration corresponding to column (1) of Table E1) and perform a comprehensive welfare analysis. From equations (68), (70) and (71) in Appendix C.3, it follows that the expanded model shares the following property with the baseline model (see footnote [19]): In a mixed finance equilibrium ($L < K$), the steady state capital stock is invariant with respect to regulation (as $R^E = 1/\beta$ in steady state). As a result, the model has immediate transitions under a mixed finance equilibrium (recall that K is the only state variable of the recursive representation). That is, starting from a capital stock that is at its steady state level, in

[79] We removed banks with total deposits below zero, banks with loans to assets ratios below 25% and banks with total asset growth above 20%.

Table E1: Calibration for the Mixed Finance Steady State

		Value (different targets)		
		Basel III	Basel III	
		d_{ss}/c_{ss}	L_{ss}/c_{ss}	
		(1)	(2)	
Parameter				*Source*
γ_{Pre}	Cap. Req. pre-Basel	0.098	0.098	
λ_{Pre}	Liq. Req. pre-Basel	0	0	
γ	Cap. Req.	0.130	0.130	
λ	Liq. Req.	0.170	0.170	
α_K	Capital share	0.30		Standard
δ	Depreciation rate	0.12		Standard
n_D	Mg. cost deposits	0.0122		Measured in section 5
n_L	Mg. cost loans	0.019	0.018	Calibrated
A	TFP	21.58	21.55	Calibrated
β	Discount factor	1/1.06		Standard
σ_u	U risk aversion	2.0		Begenau and Landvoigt (2022)
σ_v	V risk aversion	1.6		Begenau and Landvoigt (2022)
e	CES liq. pref	1.25		Begenau and Landvoigt (2022)
a	CES liq. pref	0.373	0.305	Calibrated
φ	Mixed finance cost	0.0008	0.0001	Calibrated
θ_v	Ut. coeff. liquidity	0.016	0.017	Calibrated
ψ	Resolution cost	0.32	0.32	Calibrated
σ_ϵ	S.D. credit risk	0.044	0.044	Calibrated
$F_w \sim Beta(arg1, arg2)$	Deposit withdrawal dist.	(0.18, 23.51)	(0.18, 23.51)	Calibrated
$\bar{B}$	Gov. bond supply	0.425 Y^*		Measured FRED
Data-based targets and model moments				
Y^*	100.00	100.00	100.00	Normalized
d_{ss}/c_{ss}	0.94	0.94		FDIC and NIPA
L_{ss}/c_{ss}	1.07		1.07	FDIC and NIPA
$R^D - R^B$ %	-0.81	-0.81	-0.81	Measured in section 5
$R^E - R^D$ %	3.36	3.36	3.36	Measured in section 5
L_{ss}/K_{ss}	0.42	0.42	0.50	Durdu and Zhong (2023)
p_F (pre-Basel) Basel III %	(1.07)	(1.07) 0.16	(1.07) 0.16	Corbae and D'Erasmo (2021)
Mean deposit withdrawal %	0.75	0.75	0.75	Call Reports
p99 deposit withdrawal %	8.64	8.64	8.64	Call Reports
DIF losses / deposits %	25.96	25.96	25.96	FDIC Bank Failures & Assistance
Liq. driven failures in % (Pre-Basel) Basel III		(11.8) 40.3	(11.8) 42.0	
Distortions				
τ^E		0.0017	0.0017	
τ^L		0.0001	0.0001	
$\tau^{\bar{w}}$		0.0	0.0	
Q		-0.0304	-0.0361	
Optimal Regulation				
γ^*	Optimal Cap. Req.	0.132	0.132	
λ^*	Optimal Liq. Req.	0.179	0.179	

response to a change in one or both of the regulatory requirements such that the economy stays in a mixed finance equilibrium, the capital stock is constant, and the economy immediately moves to the new steady state, where only variables other than the capital stock, such as bank lending or deposits, are potentially different. Thus, we can accurately determine the welfare-maximizing levels of capital and liquidity requirements by comparing welfare levels at steady states across various regulatory configurations. We have also verified numerically that this is correct.

Results

As shown in the last two rows of Table E1 and in Figure E1, optimal regulation is obtained at a liquidity requirement of $\lambda^* = 0.179$ and a capital requirement of $\gamma^* = 0.132$, roughly aligning with current Basel III standards ($\lambda = 0.170$ and $\gamma = 0.130$). The consumption-equivalent welfare effects relative to Basel III are negligible near current regulatory levels and remain relatively flat along the liquidity regulation dimension. However, Figure E1 reveals significant nonlinearities along the capital requirement dimension, with substantial welfare losses at lower values of γ due to increased bank failure probabilities (as shown in Figure E3). Moving from pre-Basel regulation ($\lambda_{Pre} = 0$ and $\gamma_{Pre} = 0.098$) to Basel III standards ($\lambda = 0.17$ and $\gamma = 0.13$) implied a 0.21% welfare gain in consumption-equivalent terms.

Figure E2 decomposes the marginal welfare effects of regulatory adjustments. Marginal welfare costs exhibit little nonlinearity and remain nearly constant across requirement levels, while marginal welfare benefits display pronounced nonlinearity, decreasing rapidly at low requirement levels before flattening at higher levels. Consistent with the baseline model estimates of section 5 but now incorporating benefit effects, capital regulation generates marginal welfare changes approximately an order of magnitude larger than those from liquidity regulation.

Finally, Table E2 presents a range of optimal regulatory requirement estimates derived from the model. The optimal liquidity requirements range from 0.17 to 0.24, while optimal capital requirements range from 0.12 to 0.16. These ranges reflect variations in the marginal noninterest costs of servicing deposits n_d described in section 5, alternative estimates of resolution costs (see BCBS (2010)), and different bank failure probabilities documented in the literature.

Regarding the range of resolution costs, BCBS (2010) documents that banking crises occur on average every 20 to 25 years (corresponding to an average annual crisis probability of 4.5%) and generate substantial output losses that persist beyond the crisis year. The study provides estimates based on varying assumptions about the severity and persistence of crisis effects, ranging from output losses of 19% under the assumption of no permanent effects to 63% under moderate permanent effects. These estimates serve to derive implied resolution costs consistent with the model's framework. Specifically, ψ is obtained by matching total resolution costs ($\psi p_F L$) to the expected output loss (0.045 multiplied by either 19% or 63%). This procedure yields $\psi = 1.13$ for the no permanent effect scenario and $\psi = 3.81$ for the moderate permanent effect scenario.

Regarding different bank failure probabilities, the baseline estimate of 1.07% comes from Corbae and D'Erasmo (2021), while the 0.50% estimate comes from Elenev et al. (2021). The 1.45% estimate accounts for the Troubled Asset Relief Program's (TARP) effects on observed bank failure rates. Specifically, we make two adjustments to the baseline estimates from Corbae and D'Erasmo (2021). First, we augment the historical failure rate to reflect banks that would likely have failed absent government intervention. During the calibration period, 770 depository institutions received TARP capital injections out of 8,415 institutions operating at the end of 2007. Annualizing this over the 1984-2007 period yields an additional 0.38 percentage points per year, raising the adjusted failure probability from 1.07% to 1.45%. Second, we adjust the Deposit Insurance Fund (DIF) loss-given-failure estimates by the ratio of adjusted to baseline failure rates (1.45/1.07 = 1.356). This proportional scaling is warranted given that the Capital Purchase Program ultimately generated a $16.3 billion gain, implying negligible net losses per dollar of recipient deposits relative to counterfactual failures. The baseline DIF loss rate target of 25.96% becomes 19.14% under this adjustment.

Lastly, regulation plays a meaningful role even in the absence of resolution costs (i.e., when $\psi = 0$). This regulatory role stems from the distortions created by the externalities that the social planner internalizes but individual banks do not (the values of the distortions (τ^E and τ^L) are reported for the baseline calibration in Table E1). As expected, this reduces optimal requirements. However, a significant role for regulation remains, with an implied optimal capital requirement of 5 percent and an optimal liquidity requirement of 12 percent. Thus, the distortions account for roughly half of the optimal levels of the requirements.

Figure E1: Welfare Level in Consumption-Equivalent percent relative to Basel III

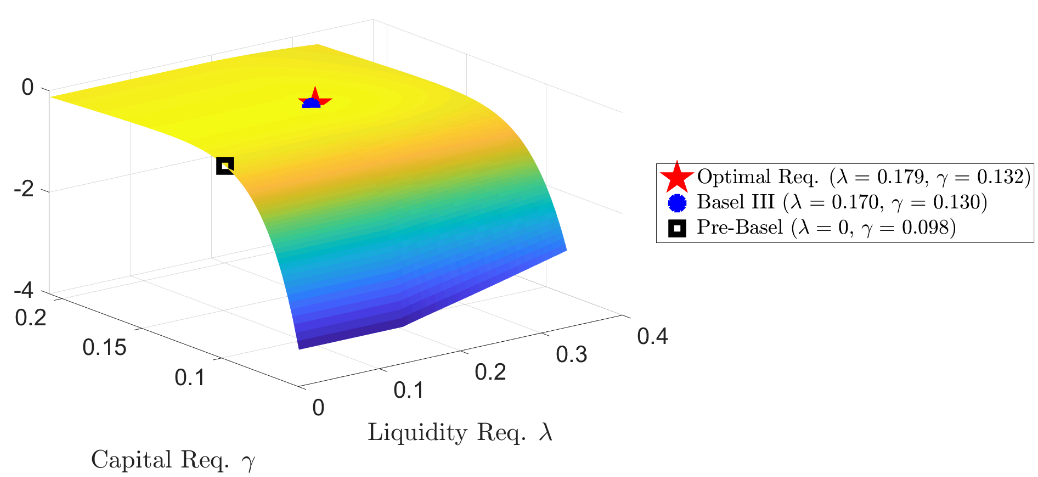
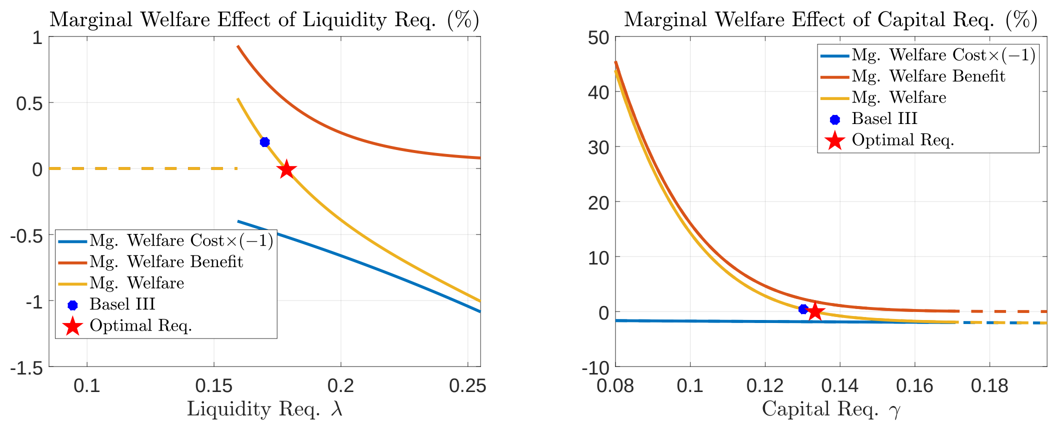
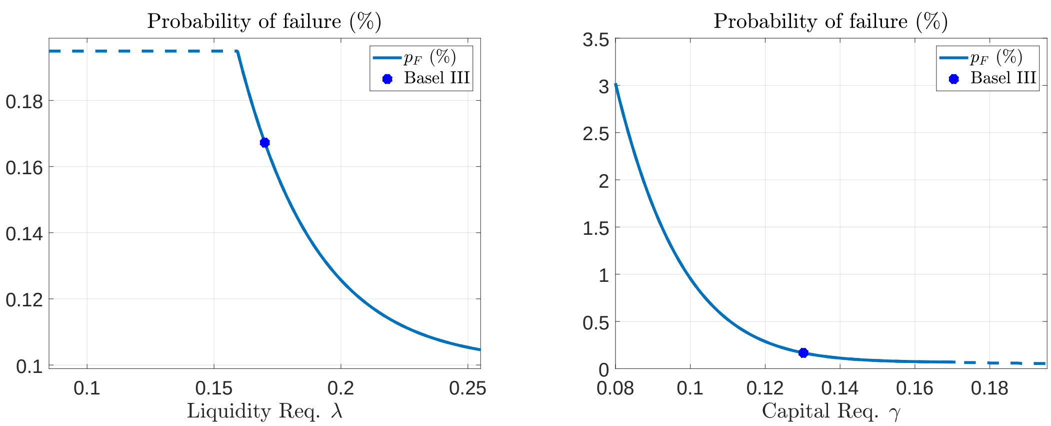


Figure E2: Marginal Welfare Decomposition



Note: Dashed lines correspond to estimates in which the liquidity constraint is slack.

Figure E3: Comparative Statics



Note: Dashed lines correspond to estimates in which the liquidity constraint is slack.

Table E2: Range of Optimal Requirements Estimates

Parameter change	Optimal Policy	
	Liq. Req. (λ^*)	Cap. Req. (γ^*)
$n_d = 0.0080$	0.207	0.130
$n_d = 0.0090$	0.198	0.131
$n_d = 0.0122$ (Baseline)	0.179	0.132
$\psi = 0$	0.116	0.045
$\psi = 0.32$ (Baseline)	0.179	0.132
$\psi = 1.13$ (BCBS (2010) no perm. eff.)	0.209	0.148
$\psi = 3.81$ (BCBS (2010) moderate perm. eff.)	0.237	0.161
$p_F = 0.50\%$ (Elenev et al. (2021))	0.181	0.120
$p_F = 1.07\%$ (Baseline)	0.179	0.132
$p_F = 1.45\%, \psi = 0.27$ (TARP recipients and cost adj. target 19.14%)	0.172	0.134

Note: Estimates are obtained using the calibration in column (1) of Table E1.

References

BCBS (2010). An assessment of the long-term economic impact of stronger capital and liquidity requirements. *L.E.I. report Bank for International Settlements August.*

Begenau, J. and T. Landvoigt (2022). Financial regulation in a quantitative model of the modern banking system. *The Review of Economic Studies 89*(4), 1748–1784.

Corbae, D. and P. D'Erasmo (2021). Capital buffers in a quantitative model of banking industry dynamics. *Econometrica 89*(6), 2975–3023.

Correia, S., S. Luck, and E. Verner (2026). Failing banks. *The Quarterly Journal of Economics 141*(1), 147–204.

Durdu, C. B. and M. Zhong (2023). Understanding bank and nonbank credit cycles: a structural exploration. *Journal of Money, Credit and Banking 55*(1), 103–142.

Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2021). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica 89*(3), 1361–1418.

Mendoza, E. G. and S. Villalvazo (2020). Fipit: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints. *Review of Economic Dynamics 37*, 81–102.

Poterba, J. M. and J. J. Rotemberg (1985). Money in the utility function: An empirical implementation. In W. A. Barnett and K. J. Singleton (Eds.), *New Approaches to Monetary Economics*, Chapter 10. Cambridge University Press.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*(2), 298–320.