Loan-to-Value Constraints and Financial Amplification: Analytical Characterization*

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Abstract

We investigate how loan-to-value (LtV) borrowing limits affect crisis severity in models with occasionally-binding collateral constraints. Analytically characterizing equilibrium and extending to stochastic environments, we uncover a U-shaped consumption response: tightening LtV constraints reduces crisis amplification when limits are low but increases amplification when limits are high. This reflects competing forces–higher limits increase leverage vulnerability but dampen asset price declines. Event studies reveal that higher LtV limits reduce crisis frequency, yet the U-shaped relationship remains present. Normative analysis shows welfare is maximized at zero debt capacity, yet transitional gains create pressure for financial deepening. Cross-country evidence provides suggestive support.

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1 Introduction

In the wake of the 1990s Sudden Stops experienced by numerous emerging economies and the economic turmoil of the 2007-2008 Global Financial Crisis, the quest for models capable of elucidating the dynamics of these crises has intensified. Pioneered by the work of Mendoza (2010), one prominent avenue has been the exploration of models featuring occasionally-binding credit constraints, both for normative and positive analysis. Central to these models is the incorporation of loan-to-value (LtV) type borrowing constraints, which cap borrowing at a fraction of the market value of asset holdings.

These models, enriched by the presence of occasionally-binding constraints, unveil a Fisherian debt-deflation mechanism. As shown by Schmitt-Grohé and Uribe (2017), in vulnerable economic states, exogenous shocks can trigger the tightening of these constraints, precipitating a cascade of effects. As asset prices plummet in response, the constraint tightens further, exacerbating deflationary pressures. This feedback loop often manifests in sharp declines in consumption, asset prices, and borrowing reversals–hallmarks of financial crises.

However, recent research challenges conventional wisdom regarding the amplification effects of collateral constraints. Schmitt-Grohé and Uribe (2021) demonstrate that collateral constraints do not necessarily amplify financial disturbances and may instead generate dynamics fundamentally different from those identified in earlier literature, including the possibility of multiple equilibria. They emphasize the inherent nonlinearity in how economies adjust under binding collateral constraints, which can give rise to two distinct equilibrium outcomes: one characterized by self-fulfilling financial crises driven by pessimistic expectations about collateral values, and another marked by underborrowing and excessive precautionary savings due to agents' fear of future financial distress.

This paper contributes to this evolving literature by rigorously characterizing the analytical solution of equilibrium asset prices in small open economies featuring loan-to-value collateral constraints. Our central finding is that the relationship between LtV limits and crisis severity is fundamentally nonlinear and nonmonotonic. Specifically, we uncover a U-shaped pattern: when the LtV limit is close to zero, tightening constraints induce smaller drops in consumption during crises. Conversely, when borrowers can leverage assets close to their full market value, tighter constraints induce larger drops in consumption during crises. We identify a critical threshold–where debt is limited to approximately 50 percent of collateral value in our baseline calibration–where this relationship reverses.

The quantitative magnitudes are substantial. Under our unique equilibrium calibration, economies with low LtV limits (where debt cannot exceed 10 percent of collateral value) experience a 9 percent reduction in consumption drop amplification when constraints tighten by 10 percentage points. In stark contrast, economies where borrowing approaches the full market value of collateral suffer a 10.5 percent *increase* in amplification from the same policy change. This asymmetry–where identical tightening is stabilizing in low-LtV regimes but destabilizing in high-LtV regimes–has profound implications for the design of macroprudential regulations.

The nonlinear impact of LtV limits reflects a general equilibrium effect operating through asset prices. When the LtV limit rises, two conflicting forces come into play. First, increased indebtedness makes the economy more fragile, amplifying the decline in consumption during crises (the *vulnerability effect*). Second, a more relaxed collateral constraint eventually mitigates the downward pressure on asset prices, dampening the severity of the crisis (the *price stabilization effect*). In economies with low LtV limits, the vulnerability effect dominates, whereas in economies with high LtV limits, the price stabilization effect prevails. This tension between leverage-induced fragility and constraint-induced price amplification generates the U-shaped relationship that is central to our analysis.

To isolate this mechanism, we extend the perfect foresight analysis to an environment with aggregate uncertainty. In this stochastic setting, we compute impulse responses under both general equilibrium (where asset prices adjust endogenously) and partial equilibrium (where asset prices are held fixed), demonstrating that the nonmonotonic effects arise specifically from the pecuniary externality operating through collateral values. We further examine the frequency and severity of endogenous financial crises through an event study. Simulating the model over a long horizon, we identify Sudden Stop episodes and document how their probability and magnitude vary systematically with the LtV limit. This analysis reveals that while higher LtV limits reduce crisis frequency by allowing greater borrowing capacity, the U-shaped consumption response persists across crisis episodes–a nonmonotonic pattern central to macroprudential policy design.

Our analysis also addresses normative questions regarding optimal LtV policy. We show that steady-state welfare is maximized when borrowers have no debt capacity–that is, when the LtV fraction limit equals zero–reflecting the overborrowing externality identified in the literature. However, transitional welfare analysis reveals a critical tension: economies experience substantial short-term gains from relaxing LtV limits, as households can better smooth consumption while accumulating debt. This creates a time-consistency problem for policymakers, who face political pressure to prioritize immediate welfare improvements over long-run efficiency. The divergence between short-run transitional gains and long-run steady-state costs helps explain the empirical regularity that countries systematically allow excessive financial deepening despite its ultimate welfare cost.

To assess the empirical plausibility of our theoretical findings, we examine cross-country evidence on how the level of financial development–which we interpret as a proxy for the model's LtV limits–correlates with the severity of consumption declines during financial crises. We construct a panel database combining Sudden Stop episodes from Bianchi and Mendoza (2020), macroeconomic aggregates from The World Bank (2024), and different financial development proxies. Quadratic regressions reveal patterns consistent with our theoretical predictions: we find a U-shaped relationship between consumption growth and financial development during crises. We emphasize that this evidence is suggestive rather than conclusive–while the empirical patterns align with our collateral constraint mechanism, alternative channels such as financial sector size effects, insurance mechanisms, or regulatory quality could generate similar nonlinear relationships. Definitively isolating the collateral constraint channel would require more targeted identification strategies, such as natural experiments around LtV policy changes or micro-level data linking actual LtV limits to household outcomes, which we view as important directions for future research. Nevertheless, the consistency between our theoretical predictions and cross-country patterns strengthens confidence that the mechanisms identified in our model are empirically relevant.

Our paper is related to several strands of the literature. The investigation of financial crises, particularly within the context of emerging economies, has been significantly advanced by models incorporating occasionally-binding credit constraints. These models, pioneered by Mendoza (2010), have provided essential insights into the mechanisms driving Sudden Stops and economic downturns. Central to this approach is the loan-to-value (LtV) constraint, which limits borrowing based on the market value of assets. Schmitt-Grohé and Uribe (2017) further developed these models by examining stock collateral constraints, revealing how exogenous shocks can tighten these constraints, leading to a Fisherian debt-deflation cycle that exacerbates financial instability.

The literature has also explored the possibility of multiple equilibria arising from these constraints. Schmitt-Grohé and Uribe (2021) analyzed flow collateral constraints, demonstrating that these models can result in nonlinear dynamics and the potential for multiple equilibria. Similarly, Jeanne and Korinek (2019) discussed the implications of stock collateral constraints, suggesting that under certain conditions, these constraints could lead to self-fulfilling financial crises or excessive precautionary savings behavior. Recently, Rojas and Saffie (2025) provide analytical characterization of amplification mechanisms in models with flow collateral constraints.

In addition to the theoretical contributions, various studies have employed these models to explore the broader implications of collateral constraints. For instance, Bianchi and Mendoza (2018) and Jeanne and Korinek (2019) applied similar models to investigate the role of pecuniary externalities in financial crises, while other scholars such as Lorenzoni (2008), Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013, 2016), and Ottonello, Perez, and Varraso (2022) examined how these constraints influence overborrowing and the design of optimal macroprudential policies. Jensen, Ravn, and Santoro (2018) studied the business-cycle implications of changing LtV limits. Devereux, Young, and Yu (2019) and Dong, Wu, and Young (2023) also contributed to this discussion by exploring the capital flow dynamics under different types of constraints and regulatory frameworks.

Our contribution differs from this literature in several important respects. First, while previous work has focused on whether collateral constraints amplify or dampen shocks, or on the existence of multiple equilibria, we characterize how the *level* of the LtV limit affects crisis dynamics. Second, we provide closed-form analytical solutions for equilibrium asset prices under perfect foresight, allowing us to derive precise conditions for uniqueness, multiplicity, and non-existence of equilibria as functions of the LtV limit and structural parameters. Third, we go beyond traditional impulse response analysis by conducting event studies that endogenize crisis occurrence and examine both frequency and severity. Fourth, our welfare analysis explicitly addresses the time-consistency problem in macroprudential policy by comparing steady-state and transitional welfare. Finally, we demonstrate that the nonlinear relationship between financial development and crisis severity is not merely a theoretical result but an empirically relevant phenomenon observable in cross-country data.

The remainder of the paper is organized as follows. Section 2 describes the Sudden Stop model with perfect foresight and characterizes its analytical solution, deriving conditions for uniqueness, multiplicity, and non-existence of equilibria. In Section 3 we extend the model to an environment with aggregate uncertainty, solving it numerically and conducting three complementary analyses: (i) impulse response functions that isolate the role of pecuniary externalities by comparing general and partial equilibrium, (ii) event studies that examine the endogenous frequency and severity of Sudden Stops across different LtV levels, and (iii) welfare comparisons that reveal the time-consistency problem inherent in macroprudential policy. Section 4 describes the model validation exercise, showing that our theoretical predictions align with cross-country evidence on financial development and crisis severity. Finally, Section 5 concludes and discusses policy implications.

2 Model with Perfect Foresight

2.1 Model Setup and Key Assumptions

We develop a small open economy model that builds on the frameworks of Korinek and Mendoza (2014), Bianchi and Mendoza (2018) and Jeanne and Korinek (2019), with several important features designed to capture the nonlinear dynamics of financial crises. The economy is populated by a continuum of homogeneous households who face borrowing constraints tied to the value of their asset holdings.[1]

2.1.1 Household Problem

The representative household maximizes expected lifetime utility with constant relative risk aversion (CRRA) preferences:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{1}$$

where $\sigma > 0$ is the coefficient of relative risk aversion. The household makes consumption and portfolio decisions subject to a budget constraint and a collateral constraint.

The household can trade two types of assets: (i) international bonds, b', that pay an exogenous risk-free interest rate R, and (ii) domestic assets, a', with an endogenously determined price q. The domestic asset is in fixed supply, normalized to unity, and generates dividend income equal to a fraction α of total output y. This specification captures the key feature that asset values are tied to the productive capacity of the economy, creating a natural link between real economic activity and financial conditions.

Total output in the economy is given by y, which is divided between labor income $(1-\alpha)y$ and capital (dividend) income αy. This decomposition is standard in the literature and allows us to capture how the importance of capital income affects the severity of financial crises. As we will show, the capital share α plays a crucial role in determining whether the economy exhibits unique

[1] Throughout the paper, we employ a recursive representation wherein the superscript $'$ denotes variables pertaining to the subsequent period, unless explicitly stated otherwise.

or multiple equilibria.

The household's borrowing is constrained by a loan-to-value (LtV) collateral constraint:

$$R^{-1}b' \geq -\kappa q a' \tag{2}$$

where $\kappa \in [0, 1]$ represents the maximum fraction of asset value that can be borrowed against. This constraint embodies the key friction in our model: lenders can only recover a fraction κ of the collateral value in case of default, limiting the amount they are willing to lend.

The recursive formulation of the household's problem is:

$$\begin{aligned} V(a, b) &= \max_{\{c,b',a'\}} u(c) + \beta V(a', b') \quad \text{s.t.} \\ c + R^{-1}b' + qa' &= (1-\alpha)y + a(\alpha y + q) + b \\ R^{-1}b' &\geq -\kappa q a'. \end{aligned} \tag{3}$$

Timing and Enforcement Assumptions. The timing of the collateral constraint in equation (2) deserves careful discussion, as it has important implications for the model's dynamics. We use the current asset price q and next period's asset holdings a' as collateral, reflecting a setting where: (i) credit contracts are written based on the current market value of assets that will serve as collateral in the next period; (ii) lenders can seize and liquidate assets at the prevailing market price if borrowers default; and (iii) the fraction κ captures both legal/institutional factors affecting creditor rights and potential fire-sale discounts in liquidation.

This timing assumption, while standard in much of the literature (Mendoza, 2010; Korinek and Mendoza, 2014; Schmitt-Grohé and Uribe, 2017; Jeanne and Korinek, 2019), differs from alternative specifications that use current-period asset values (Bianchi and Mendoza, 2018), expected asset values (Devereux, Young, and Yu, 2019) or flow collateral constraints (Benigno, Chen, Otrok, Rebucci, and Young, 2013, 2016; Schmitt-Grohé and Uribe, 2021; Rojas and Saffie, 2025). In Section 2.4 below, we discuss how our results would change under these alternative timing assumptions.

2.1.2 Equilibrium Conditions

Let $\lambda > 0$ denote the Lagrange multiplier on the budget constraint and $\mu \geq 0$ the multiplier on the collateral constraint. Define $\hat{\mu} = \mu/\lambda \geq 0$ as the shadow value of relaxing the collateral constraint in consumption units. The first-order conditions, combined with the market clearing condition $a = a' = 1$, yield:

$$u_c(c) = R\beta u_c(c') + \mu \tag{4}$$

$$q = \beta \frac{u_c(c')(\alpha y' + q')}{u_c(c)(1 - \kappa\hat{\mu})} \tag{5}$$

$$0 = \mu(R^{-1}b' + \kappa q) \tag{6}$$

$$c = y + b - R^{-1}b'. \tag{7}$$

Equation (4) is the Euler equation for bond holdings, showing that when the collateral constraint binds ($\mu > 0$), the household's marginal utility of consumption exceeds the discounted expected marginal utility from saving, reflecting the shadow cost of the borrowing constraint.

Equation (5) is the forward-looking asset pricing equation, which reveals the central mechanism of our model. The asset price depends on the expected future payoff $(\alpha y' + q')$ discounted by the stochastic discount factor $\beta u_c(c')/u_c(c)$, but is amplified by the term $(1 - \kappa\hat{\mu})^{-1}$ when the constraint binds. This amplification effect captures how binding borrowing constraints inflate asset prices above their fundamental value, creating the potential for sharp price declines when constraints tighten.

2.1.3 The Equity Premium and the Role of LtV Limits

Combining equations (4) and (5), we can derive an expression for the equity premium, defined as the difference between the return on the domestic asset $R^q = (\alpha y' + q')/q$ and the risk-free rate R:

$$R^q - R = \underbrace{R\left(\frac{\hat{\mu}}{1-\hat{\mu}}\right)}_{\text{Indirect Effect}} \underbrace{(1-\kappa).}_{\text{Direct Effect}} \tag{8}$$

This decomposition reveals two channels through which the LtV limit κ affects the equity premium:

Direct Effect. The term $(1-\kappa)$ shows that, holding the shadow value $\hat{\mu}$ constant, a higher LtV limit (larger κ) mechanically reduces the equity premium. This occurs because looser borrowing constraints allow households to lever up more easily, reducing the compensation they require for holding the risky asset. When $\kappa = 1$ this direct effect vanishes entirely. Intuitively, if households can borrow the full value of their assets, they face no leverage constraint and the equity premium becomes zero.

Indirect Effect. The term $\hat{\mu}/(1-\hat{\mu})$ captures how the LtV limit affects the tightness of the constraint in equilibrium. This effect operates through general equilibrium and can work in either direction. In economies with low initial LtV limits, increasing the borrowing fraction allows more debt accumulation, which can make the constraint bind more tightly in crisis states (higher $\hat{\mu}$), potentially increasing the equity premium through this channel. Conversely, in economies with high initial LtV limits, the constraint is already relatively loose, so further increases have diminishing effects on $\hat{\mu}$.

The interaction between these direct and indirect effects generates the nonmonotonic relationship between LtV limits and crisis severity that is central to our analysis. This connection to empirical equity premium puzzles is particularly relevant: our model suggests that observed equity premia may reflect not just fundamental risk, but also the shadow cost of financial constraints, which varies systematically with the level of financial development.

2.2 Stationary Equilibrium

We now characterize the stationary equilibrium under the assumption that $R\beta = 1$, which ensures that consumption remains constant in steady state: $c = c'$. It is well known that without any collateral constraint, the steady state becomes a function of initial bond holdings b. This property also holds as long as $b > -R\kappa\frac{\beta\alpha y}{1-\beta}$. Using equations (4)–(7), the unconstrained steady state is characterized by:

$$c = y + (1-\beta)b \tag{9}$$

$$q = \frac{\beta\alpha y}{1-\beta}. \tag{10}$$

The household maintains constant consumption equal to total endowment y plus the interest earned or paid depending on the sign of the initial bond position b. The current account is constant and zero ($CA = b' - b = 0$), the collateral constraint does not bind ($\mu = 0$), and the equilibrium exhibits the standard forward-looking asset price, which is independent of the LtV limit.

When the household is marginally constrained, meaning the debt is at its highest level but the collateral constraint multiplier is zero ($b = -R\kappa q$ and $\mu = 0$), the equilibrium allocations become:

$$b = b' = -\frac{\kappa\alpha y}{1-\beta} \tag{11}$$

$$c = y(1-\kappa\alpha). \tag{12}$$

These expressions reveal how the LtV limit affects steady-state outcomes. Higher values of κ allow for greater borrowing (more negative b), which in turn reduces steady-state consumption through the debt service burden. The magnitude of this effect depends on the capital share α: economies with higher capital income can support more debt relative to output, but also face larger consumption losses when highly leveraged. This trade-off between borrowing capacity and consumption smoothing is central to understanding the welfare implications of different LtV limits, which we explore in detail in Section 3.

2.3 Financial Crisis Analytical Results

The analytical results in this section provide a clear and concise derivation of the equilibrium allocations and asset prices in a simplified model setup with a logarithmic utility function (the coefficient of risk aversion is equal to 1). By assuming an unexpected wealth-neutral shock to the economy (see Eq. (13) for a sequential representation of the shock), we can explore how the LtV limit influences the outcomes of the model under different scenarios. This shock generates a financial crisis because, in the absence of the collateral constraint, such a wealth-neutral shock would have no effect on consumption. The economy would be able to perfectly smooth consumption by adjusting the current account. During period 0, the economy would borrow from abroad and from period 1 onward, the economy would pay the interest on such borrowing with the additional endowment.

$$y_t = \begin{cases} y & \text{for } t \leq -1 \\ \gamma y & \text{for } t = 0 \text{ with } \gamma < 1 \\ \tilde{y} = y(1 + \left(\frac{1-\beta}{\beta}\right)(1-\gamma)) & \text{for } t \geq 1. \end{cases} \tag{13}$$

We assume that the economy is marginally constrained and the initial bond holding is at the debt limit, $b = -R\kappa q$. It is worth noting that this assumption implicitly changes the initial condition whenever the LtV limit changes. This is needed because, as noted earlier, the only source of pass-through of the shock to consumption comes from the collateral constraint. A similar assumption is used by Rojas and Saffie (2025). Alternatively, following Schmitt-Grohé and Uribe (2017), we could assume an initial unexpected shock that takes the economy to the debt limit, followed by the shock from Eq. (13). Another approach would be to assume $R\beta < 1$, in which case the economy would converge to a constrained steady state in the long run, though this would preclude analytical results. Section 3.1.1 shows an impulse response analysis which numerically validates the following analytical results in a more general framework with $R\beta < 1$ and stochastic endowment.

The following equations summarize the equilibrium allocations and asset prices around the financial crisis that occurs at $t = 0$:

$$q_{-1} = \frac{\beta\alpha y}{1-\beta} \tag{14}$$

$$c_{-1} = y(1-\kappa\alpha) \tag{15}$$

$$b_0 = -\frac{\kappa\alpha y}{1-\beta} \tag{16}$$

$$c_0 = \gamma y + b_0 - R^{-1}b_1 = y\left(\frac{\gamma(1-\beta)-\kappa\alpha}{1-\beta}\right) + \kappa q_0 \tag{17}$$

$$b_1 = -R\kappa q_0 = b_\tau \text{ for } \tau \geq 2 \tag{18}$$

$$c_1 = \tilde{y} + b_1 - R^{-1}b_2 = \tilde{y} - \frac{1-\beta}{\beta}\kappa q_0 = c_\tau \text{ for } \tau \geq 2 \tag{19}$$

$$q_1 = \frac{\beta\alpha\tilde{y}}{1-\beta} = q_\tau \text{ for } \tau \geq 2. \tag{20}$$

Note that the only unknown is the current price q_0. Combining Eq. (4)–(5) and substituting Eq. (17)–(20), we obtain a quadratic polynomial:

$$q_0 = \beta\frac{c_1^{-1}(\alpha\tilde{y} + q_1)}{c_0^{-1}(1-\kappa\hat{\mu}_0)}$$

$$\Leftrightarrow$$

$$q_0 = \left(\frac{\beta\alpha\tilde{y}}{1-\beta}\right)\frac{y\left(\frac{\gamma(1-\beta)-\kappa\alpha}{1-\beta}\right) + \kappa q_0}{\kappa\left(y\left(\frac{\gamma(1-\beta)-\kappa\alpha}{1-\beta}\right) + \kappa q_0\right) + (1-\kappa)\left(\tilde{y} - \frac{1-\beta}{\beta}\kappa q_0\right)}$$

$$\Leftrightarrow$$

$$0 = Aq_0^2 + Bq_0 + C.$$

Where:

$$
\begin{aligned}
A &= \frac{\kappa}{\beta}(\kappa + \beta - 1) \\
B &= \kappa y \gamma \left(1 - \frac{\kappa\alpha}{\gamma(1-\beta)}\right) + \tilde{y}\left(1 - \kappa - \frac{\beta\alpha\kappa}{1-\beta}\right) \\
C &= \frac{\beta\alpha\tilde{y}y\gamma}{1-\beta}\left(\frac{\kappa\alpha}{\gamma(1-\beta)} - 1\right).
\end{aligned}
$$

Which can be solved using the general quadratic formula solution:

$$
q_0^H = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad q_0^L = \frac{-B - \sqrt{B^2 - 4AC}}{2A}. \tag{21}
$$

Equation (21) is the analytical solution for the asset price in the period when the unexpected wealth-neutral shock hits the economy. In the next subsection, we use this analytical solution to characterize the parameter regions where the equilibrium is unique, multiple, or non-existent.

2.3.1 Characterization of the Equilibrium Asset Price and Cases

In this subsection, we obtain the general cases for uniqueness, multiplicity, or non-existence of the equilibrium.

Assumption 1. *The drop in the endowment is not too large:* $\gamma > \frac{\alpha}{1-\beta(1-\alpha)}$.

Assumption 1 is plausible since γ corresponds to 1 minus the size of the shock, so it is expected to be close to 1, and α corresponds to the capital income share, which is expected to be less than 0.33. Now we obtain the following cases:

Cases:

$$
\text{Unique}\begin{cases}
A=0 \Leftrightarrow (\kappa = 1-\beta \text{ or } \kappa=0) \text{ then } q_0 = \frac{-C}{B} > 0 \text{ since } B>0, C<0. \\[2ex]
A>0 \text{ and } C<0 \Leftrightarrow \kappa \in [1-\beta, \min\{1,\ (1-\beta)\frac{\gamma}{\alpha}\}) \text{ then } q_0^H > 0. \\[2ex]
A<0 \Leftrightarrow \kappa < 1-\beta \text{ then} \\[2ex]
\quad q_0^H > 0,\ q_0^L > 0 \text{ but only } q_0^H > 0 \text{ is consistent with } c_0 \le c_{-1}. \\[2ex]
\kappa = 1 \text{ then } q_0 = \frac{\beta\alpha\tilde{y}}{1-\beta}. \\[2ex]
\kappa = (1-\beta)\frac{\gamma}{\alpha} \text{ then } q_0 = \frac{-B}{A} > 0 \text{ since } B<0, A>0.
\end{cases}
$$

$$
\text{Multiple (pair)}\begin{cases} A>0\,, C>0 \text{ and } B^2-4AC>0 \text{ then } q_0^H>0,\ q_0^L>0. \end{cases}
$$

$$
\text{Non-existence}\begin{cases} B^2-4AC<0. \end{cases}
$$

In summary, $0 \le \kappa \le \min\{1,\ (1-\beta)\frac{\gamma}{\alpha}\}$ guarantees a unique equilibrium. Intuitively, more volatile economies (lower γ) and economies with higher capital returns (higher α) require a lower LtV limit to guarantee a unique equilibrium. This result is important since it says that a unique equilibrium is guaranteed to exist when the LtV limit is close to zero.

Having a closed-form solution for the equilibrium asset price allows us to establish the following results.

Proposition 1. *The economy has a unique equilibrium if and only if the LtV limit is below the discounted capital share adjusted for the shock:* $0 \le \kappa \le \min\{1,\ (1-\beta)\frac{\gamma}{\alpha}\}$*. If* $\kappa > (1-\beta)\frac{\gamma}{\alpha}$ *then there is multiplicity when* $B^2-4AC>0$ *and non-existence when* $B^2-4AC<0$*.*

To obtain this result, we rely on the properties and cases of the quadratic solution of the asset price presented in Eq. (21). QED.

Lemma 1. *The economy has a unique equilibrium if and only if the debt-to-endowment ratio is less than or equal to the shock size:* $\frac{-b}{y} \le \gamma$*. In the special case of an arbitrarily small shock (*$\gamma \approx 1$*), a unique equilibrium is guaranteed when the debt-to-endowment ratio is below 100 percent.*

To obtain this result, we use the bond expression in Eq. (11) divided by the endowment and note that under a unique equilibrium, $\alpha \leq (1-\beta)\frac{\gamma}{\kappa}$. Combining the two expressions delivers the *if* part of Lemma 1. To obtain the *only if* part, we start from the debt-to-endowment ratio $\left(\frac{-b}{y}\right)$ and replace the bond expression in Eq. (11). After some manipulation we obtain the uniqueness condition. QED.

Proposition 2. *When debt is not possible ($\kappa = 0$) then:*

- *One-to-one mapping from shock to drop in price and consumption: $\frac{q_0}{q_{-1}} = \frac{c_0}{c_{-1}} = \gamma$.*
- *Crisis amplification or dampening from increases in the LtV from zero depend on the capital share for the asset price and unambiguous crisis amplification for drop in consumption:*

$$\left.\frac{dq_0}{d\kappa}\right|_{\kappa=0} = \begin{cases} \geq 0 \textit{ if } \alpha \leq (1-\beta)\gamma \\ < 0 \textit{ if } \alpha > (1-\beta)\gamma \end{cases}, \quad \left.\frac{dc_0/c_{-1}}{d\kappa}\right|_{\kappa=0} < 0.$$

The first part of this result is obtained by substituting $\kappa = 0$ in Eq. (14)–(17) and (21). The second part is obtained after differentiating Eq. (21) with respect to κ and evaluating it at $\kappa = 0$. QED.

Proposition 3. *When the LtV limit is at the maximum and the equilibrium is unique ($\kappa = 1$ and $\alpha = (1-\beta)\gamma$) then:*

- *One-to-one mapping from shock to drop in consumption, $\frac{c_0}{c_{-1}} = \gamma$, and there is a slight price inflation, $\frac{q_0}{q_{-1}} = 1 + \left(\frac{1-\beta}{\beta}\right)(1-\gamma) > 1$.*
- *Crisis amplification from decreases in the LtV from one in both the asset price and consumption:*

$$\left.\frac{dq_0}{d\kappa}\right|_{\kappa=1} > 0, \quad \left.\frac{dc_0/c_{-1}}{d\kappa}\right|_{\kappa=1} > 0.$$

The proof of this result is analogous to how we obtained Proposition 2, evaluating the derivatives at $\kappa = 1$ and $\alpha = (1-\beta)\gamma$. QED.

Proposition 4. *When $\alpha = (1-\beta)\gamma$ and hence the equilibrium is unique over all $\kappa \in [0,1]$, then price deflation happens when $\kappa < \frac{1-\gamma(1-\beta)}{1-\gamma(1-\beta)^2}$ and inflation when κ is above such threshold.*

This result is obtained by substituting $\alpha = (1-\beta)\gamma$ and comparing q_{-1} with q_0 from Eq. (21). Since q_0 is increasing in κ (Proposition 3), we solve for the κ such that $\frac{q_0}{q_{-1}} = 1$ to obtain the threshold. QED.

The derived propositions elucidate the implications of the model. The finding that the debt-to-endowment ratio is constrained by the shock size under a unique equilibrium underscores the importance of managing leverage in the economy. Moreover, the analysis shows that the effects of increasing the LtV limit are nuanced: while it can dampen crises in some cases, it may amplify them in others, depending on the capital share and current LtV level.

This nonmonotonic behavior is largely driven by the interaction between asset prices and collateral constraints (see Eq. (8)). Specifically, when the LtV limit increases, two opposing forces come into play. On one hand, higher LtV limits allow for greater indebtedness, making the economy more vulnerable to shocks and amplifying the drop in consumption. On the other hand, a looser collateral constraint mitigates the downward pressure on asset prices, dampening the overall impact of the shock. The former effect dominates in low LtV economies, while the latter effect dominates in high LtV economies. This result emphasizes the trade-offs inherent in financial regulation and the need for careful calibration of LtV limits to balance economic growth and financial stability.

Overall, this section demonstrates the power of analytical modeling in uncovering the complex interplay between collateral constraints, asset prices, and macroeconomic outcomes. The insights gained from the closed-form solutions provide a solid foundation for understanding the broader dynamics explored in the more complex, stochastic version of the model.

2.4 Discussion on Alternative Timings in the Collateral Constraint

The specification of collateral constraints in models with occasionally-binding borrowing limits involves two key timing decisions: (i) whether to value collateral at current or future prices, and (ii) whether to use current stock or future choice of asset holdings. Our baseline specification in equation (2) employs current prices (q) and future asset holdings (a'):

$$R^{-1}b' \geq -\kappa q a'. \tag{22}$$

This subsection discusses three alternative timing specifications and explains why our choice is appropriate for studying the nonlinear effects of LtV limits.

We will start with the debt limit being equal to a fraction of the current prices and current asset holdings: $\kappa q a$. Under this collateral constraint, equilibrium conditions expressed in Eqs. (4), (6) and (7) are the same while Eq. (5) becomes

$$q = \beta \frac{u_c(c')(\alpha y' + q'(1 + \hat{\mu}'\kappa))}{u_c(c)}. \tag{23}$$

By comparing Eq. (23) with Eq. (5), we can see that now instead of the current collateral constraint multiplier, it is the next period's multiplier the one that affects the equilibrium asset price. However, under a deterministic transition to a steady state, future multipliers are zero, and the equity premium collapses to

$$R^q - R = R\left(\frac{\hat{\mu}}{1 - \hat{\mu}}\right), \tag{24}$$

where we can see that only the indirect effect of the LtV coming through the multiplier enters the equity premium. Under this timing, increasing the LtV only amplifies the declines in asset prices and consumption because now there is no endogenous general equilibrium effect that makes asset prices adjust less for high LtVs. Regarding the uniqueness of the solution, numerically we verified that under this timing, the uniqueness condition in Proposition 1 holds but, as Eq. (24)

shows, increasing the LtV would only lead to stronger amplification of the decline in asset prices, and hence consumption. Lastly, it is worth noting that, because of the stationary deterministic environment, using future prices and the current asset holdings as the debt limit in the collateral constraint, $\kappa q' a$, would collapse to the same results.

Regarding future prices and future asset holdings as debt limit, $\kappa q' a'$, again, equilibrium conditions expressed in Eqs. (4) and (7) are the same while Eq. (5) becomes

$$q = \beta \frac{u_c(c')(\alpha y' + q')}{u_c(c)} + \hat{\mu} \kappa q'. \tag{25}$$

By comparing Eq. (25) with Eq. (5), we can see that now instead of the current collateral constraint multiplier impacting the current price in a multiplicative way, it enters in an additive way. This difference has no impact on the main results of Section 2.3, which we verified numerically. Note that the equity premium now becomes

$$R^q - R = R \left(\frac{\hat{\mu}}{1 - \hat{\mu}} \right) \left(1 - \frac{\kappa q'}{q} \right), \tag{26}$$

where we can see that again, the direct effect of the LtV limit reduces the equity premium, similar to our baseline timing framework.

The next subsection discusses why our timing specification choice is appropriate for studying the nonlinear effects of LtV limit.

2.4.1 Why Our Specification is Appropriate

We use a collateral constraint timing that depends on current asset prices and future asset holdings. Specifically, borrowing is constrained by the current market value q of assets that the borrower commits to hold in the next period a'. This timing assumption follows the approach microfounded and adopted in influential work by Mendoza (2010), Korinek and Mendoza (2014), Schmitt-Grohé and Uribe (2017), Jeanne and Korinek (2019), and others in the macroprudential policy literature. Moreover, our choice is based on analytical tractability, empirical validity, and economic realism.

First, this timing specification permits analytical characterization of equilibrium dynamics and closed-form solutions for the equilibrium asset price, allocations and the equity premium during a financial crisis. As demonstrated in the previous sections, we can derive explicit expressions showing how the LtV ratio affects asset prices through both direct and indirect channels. This transparency is essential for isolating the competing mechanisms that generate the nonlinear relationship between financial leverage and asset price volatility. Second, and perhaps most importantly, our specification generates predictions that align with empirical patterns observed in the data. As we document in Section 4 on empirical validation, the data exhibit a nonmonotonic relationship between LtV limits and consumption dynamics–precisely the pattern that emerges from our theoretical framework. Alternative timing specifications, such as the current-period asset value approach discussed above, produce monotonic amplification effects that are inconsistent with these empirical regularities.

Beyond these technical considerations, our timing assumption reflects institutional features of actual credit markets. Credit contracts are typically written based on the current market valuation of assets that the borrower commits to hold as collateral in the next period. This captures common lending practices where lenders assess collateral value at prevailing market prices when extending credit, borrowers commit to maintaining a certain asset position (a') as security for the loan, and the constraint prevents simultaneous increasing leverage and asset sales. A loan officer evaluating a mortgage application uses today's property appraisal, not a forecast of next year's value. Similarly, the borrower pledges specific assets that will serve as collateral going forward. By tying borrowing capacity to committed future holdings rather than current holdings, the specification rules out borrowers immediately liquidating their collateral after obtaining credit. This structure mirrors the typical sequence in secured lending: the lender evaluates collateral worth today, the borrower receives credit based on this assessment, and the borrower maintains the assets as security until the debt is repaid.

Our timing specification also aligns closely with how macroprudential regulations are implemented in practice (Galati and Moessner, 2013; Cerutti, Claessens, and Laeven, 2017). Regulatory

LtV limits typically specify maximum loan amounts based on current appraised values of collateral, not forecasts of future values. This approach is ubiquitous across different forms of collateralized lending. Mortgage LtV limits are set by banking regulations worldwide that cap mortgage loans as a percentage of current property appraisals. For example, a borrower purchasing a home appraised at q today can borrow at most κq, regardless of expected future price appreciation. Similarly, margin requirements in securities lending and derivatives trading employ mark-to-market valuations, where margin calls are triggered when current portfolio values fall below threshold levels, not when future values are forecast to decline. Reserve requirements in banking regulations also specify reserve and capital requirements based on current book values of assets and liabilities, not projected future values. By adopting a timing convention that mirrors regulatory practice, our framework provides direct insights for macroprudential policy design, and the nonlinear effects we identify have immediate implications for optimal LtV calibration.

While alternative timing specifications generate quantitatively different dynamics, our choice of current prices with future asset holdings (q, a') offers the best combination of analytical tractability, economic interpretability, and policy relevance for studying nonlinear effects of LtV limits. The specification not only facilitates theoretical analysis but also captures key institutional features of credit markets and aligns with the structure of actual macroprudential regulations.

2.5 Numerical Illustration

To further dissect the mechanics of our model, we analyze numerically our framework, allowing us to isolate the core dynamics at play. This approach helps in understanding the fundamental behavior of the model under different levels of the loan-to-value (LtV) limit, κ, particularly focusing on the existence and nature of equilibria.

The parameters used for the numerical exercise are shown in Table 1. These follow Durdu, Mendoza, and Terrones (2009) and are standard values in the literature that studies Sudden Stops in emerging economies. A discount factor of 0.94 would imply an annual interest rate of 6.3 percent. The total endowment is normalized to one. The magnitude of the shock corresponds to a 5 percent drop in the endowment. Regarding the capital share, α, we will use three values to illustrate the different cases of the solution.

Table 1: Parameter Values

Parameter		Value
β, $(R = \beta^{-1})$	Discount factor	0.94
y	Total endowment	1.0
γ	Drop in endowment	0.95
α	Capital share	$\in \{(1-\beta)\gamma = 0.057,\ 0.15,\ 0.20\}$
κ	LtV limit	$\in [0,\ 1]$

While the capital share of $\alpha = 0.057$ in our baseline case may appear low relative to the standard calibration of $\alpha \approx 0.33$ used in business cycle models, this choice is consistent with the sudden stop literature employing stock collateral constraints. Three considerations justify this calibration. First, the α in our model represents only the dividend-generating capital that serves as collateral, not the entire capital stock. Much physical capital (infrastructure, non-tradable sector capital) cannot serve as collateral in international borrowing. Second, studies examining sudden stops with similar constraints use comparable values: Jeanne and Korinek (2019) use $\alpha = 0.05$, Bianchi and Mendoza (2018) use $\alpha = 0.03$, and Korinek and Mendoza (2014) employ $\alpha = 0.06$. Third, and most importantly, higher values of α lead to multiple or non-existent equilibria (as shown in Figures 3-4), preventing meaningful quantitative analysis across the full range of $\kappa \in [0, 1]$. Our choice maximizes analytical tractability while remaining empirically grounded in the relevant literature.

The results presented in the following figures highlight three distinct scenarios: cases where the model yields a unique equilibrium, multiple equilibria, or no equilibrium at all. These scenarios are critical for understanding the potential stability or instability of the economy under varying financial constraints.

Unique Equilibrium Case

Figure 1 specifically illustrates the case where the capital share α, is equal to $(1-\beta)\gamma$, the maximum value that guarantees a unique equilibrium across all LtV limits ranging from 0 to 1. This particular setting allows us to observe how the economy responds when the LtV limit is varied in a controlled, predictable environment, free from the complexities introduced by uncertainty.

Panel a) of Figure 1 demonstrates a monotonic dampening in the decline of asset prices as the LtV limit increases. This behavior aligns with the intuition that higher LtV limits, by loosening borrowing constraints, mitigate the downward pressure on asset prices. As the collateral constraint becomes less binding, the market is able to absorb shocks more efficiently, preventing a sharp drop in asset values.

Panel b) shows a U-shaped response in the consumption drop as the LtV limit increases. This nonmonotonic pattern is particularly noteworthy as it reflects the delicate balance between increased borrowing capacity and heightened vulnerability. At lower LtV limits, the economy is less leveraged, and the impact of shocks on consumption is relatively contained. However, as the LtV limit increases, the increased leverage makes the economy more susceptible to shocks, leading to a more pronounced decline in consumption. Beyond a certain point, though, the loosening of constraints allows for greater consumption smoothing, hence the U-shaped pattern.

Panel c) presents the corresponding change in the current account to GDP ratio, which exhibits a mirror image of the U-shaped consumption response. This inverse relationship highlights the trade-offs faced by the economy as it adjusts to shocks under different LtV regimes. When the LtV limit is low, the limited borrowing capacity leads to smaller swings in the current account. As the LtV increases, the economy finances consumption by borrowing more and when the economy becomes overly leveraged, the need to correct this imbalance in the face of adverse shocks results in a sharp reversal, hence the mirrored U-shape.

These findings from the unique equilibrium case highlight the critical role that collateral constraints play in shaping the economy's response to shocks. The existence of a unique equilibrium in the setting here indicates a predictable and stable economic environment where policy interventions can be more effectively tailored. The U-shaped consumption response and the mirrored current account behavior further emphasize the nonlinear effects of varying LtV limits, suggesting that while increasing the LtV limit can offer short-term benefits in terms of consumption smoothing, it also introduces longer-term risks associated with higher leverage.

Figure 1: Percent changes during crisis for low capital share (α)

(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

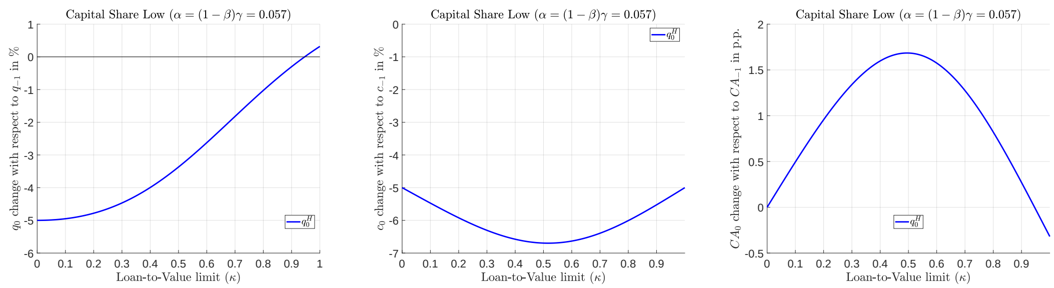
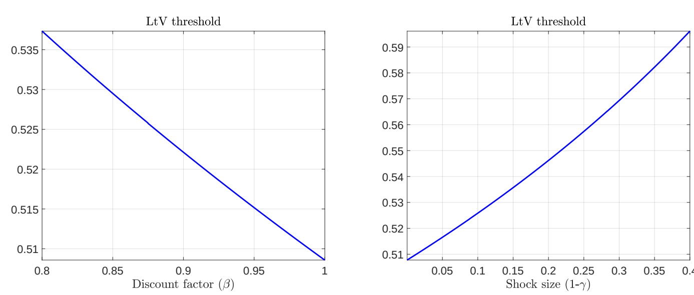


Figure 2 illustrates how the threshold on the LtV limit, which determines the minimum of the U-shaped consumption response, changes for different values of the discount factor and the size of the shock. For LtVs below (above) the threshold, increases in the LtV amplify (dampen) the decline in consumption. Panel a) shows that in more impatient economies the LtV threshold is higher. This means that the interval in which increasing the LtV amplifies the decline in consumption is larger. Similarly, panel b) shows that the same happens in economies with larger shocks.

Figure 2: Comparative Statics

(a) Discount Factor (b) Shock Size



We now explore the behavior of the model when the capital share (α) is set to medium and high levels, specifically 0.15 and 0.20, as illustrated in Figures 3 and 4, respectively. These scenarios allow us to examine how increasing the capital share influences the stability of the economy, particularly in terms of the existence and multiplicity of equilibria.

Multiple Equilibria

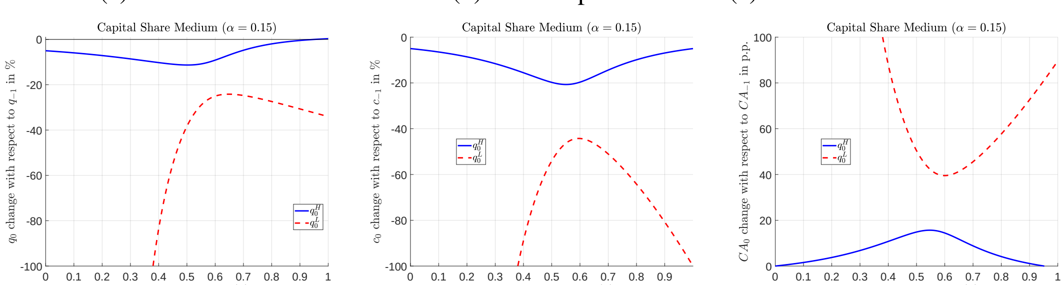
In Figure 3, where α is set to 0.15, the model begins to exhibit more complex dynamics compared to the unique equilibrium case. At this medium capital share level, certain values of the LtV limit lead to multiple equilibria. The solid blue lines correspond to the equilibrium obtained with the “high” asset price while the dashed red lines correspond to the equilibrium with the “low” asset price from Eq. 21. Note that the dynamics of the “high” asset price are similar to the dynamics under a unique equilibrium and are more stable relative to the “low” asset price, which generates total asset price collapses for some LtV limits.

This outcome reflects the increased role of capital in the production process, which amplifies the feedback effects between asset prices and collateral constraints. As the capital share rises, the value of collateral becomes more sensitive to fluctuations in asset prices, making the economy more prone to nonlinear and potentially unstable responses to shocks.

In this scenario, the economy can settle into different equilibrium states depending on the equilibrium selection device or exogenous shocks. For instance, one equilibrium might be characterized by relatively stable asset prices and consumption, while another might involve more pronounced declines in these variables. This multiplicity of equilibria introduces an element of unpredictability, as small perturbations can push the economy from one equilibrium to another, potentially triggering a financial crisis or exacerbating its effects.

Figure 3: Percent changes during crisis for medium capital share (α)

(a) Asset Price (b) Consumption (c) Current Account to GDP ratio



Non-Existent Equilibrium

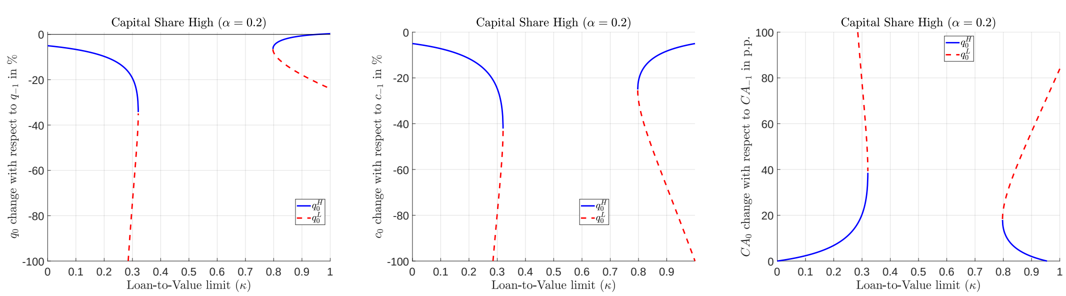
For higher levels of the capital share, there could be values of the LtV limit for which there is no solution and the equilibrium fails to exists, reflecting an extreme form of financial fragility. Figure 4 further explores this dynamic by increasing the capital share to 0.20. At this elevated level, the economy becomes even more sensitive to changes in the LtV limit, with a broader range of LtV values leading to multiple equilibria or non-existent equilibria.

The high capital share scenario underscores the dangers of high leverage in an economy where capital plays a dominant role. As α increases, the productive capacity of the economy becomes more reliant on capital, and thus, more exposed to fluctuations in asset prices. The tightening of collateral constraints in response to declining asset values can quickly spiral out of control, leading to severe disruptions in consumption and external balances.

In these high α scenarios, the model demonstrates how elevated capital intensity can exacerbate financial instability, particularly in the presence of loose borrowing constraints (high LtV limits where $\kappa > (1-\beta)\frac{\gamma}{\alpha}$). The combination of high leverage and a significant capital share creates conditions where financial markets are vulnerable, resulting in a collapse of asset prices, a sharp contraction in consumption, and potentially, a failure of the economy to reach an equilibrium.

Figure 4: Percent changes during crisis for high capital share (α)

(a) Asset Price (b) Consumption (c) Current Account to GDP ratio



The analysis of the model under medium and high capital share levels provides critical insights into the conditions under which financial markets may become unstable. The presence of multiple equilibria and the potential for non-existent equilibria highlight the risks associated with high leverage and significant capital intensity in the economy.

From a policy perspective, these findings suggest that regulators need to be particularly cautious when allowing high LtV limits in economies with a high capital share. While higher LtV limits can facilitate investment and growth during stable periods, they can also lead to multiple and potentially unstable equilibria, increasing the risk of financial crises.

In economies where capital plays a major role, policies that aim to moderate leverage and ensure more robust collateral frameworks could help prevent the onset of these unstable equilibria. Additionally, understanding the conditions that lead to non-existent equilibria can inform the design of safety nets and intervention mechanisms that could prevent the economy from spiraling into a state where markets fail to clear altogether.

Overall, the findings from these scenarios underscore the importance of considering the interaction between capital intensity and financial constraints in macroeconomic models. By highlighting the potential for multiple or non-existent equilibria, this analysis adds an important dimension to our understanding of financial stability and the factors that can precipitate or mitigate financial crises. This analysis also reinforces the importance of considering nonlinear dynamics in policy design. Policymakers need to be aware that increasing financial flexibility through higher LtV limits may only be beneficial up to a point. Beyond that, the increased vulnerability can lead to more severe downturns in the event of adverse shocks. The insights gained from this simplified model without uncertainty provide a foundational understanding that complements the more complex dynamics observed in the stochastic version of the model, which we present in the next section.

3 Model with Aggregate Uncertainty

There is not much we can do analytically when there is uncertainty in the economy. However, in this section we compute an extension of the model where the income endowment, y, is stochastic and follows an AR(1) Markov process. Specifically,

$$\log y_t = (1-\rho)\log(y_{ss}) + \rho\log(y_{t-1}) + \sigma_\epsilon \epsilon_t, \tag{27}$$

where $\epsilon_t \sim N(0,1)$. The additional parameters will take the values $\rho = 0.70$ and $\sigma_\epsilon = 0.025$, which are common values used in the literature (see Durdu, Mendoza, and Terrones (2009)). The rest of the parameters stay the same as in Table 1 with the exception of the interest rate which is lowered such that $R\beta < 1$ to guarantee a non-degenerate limiting wealth distribution: $R = 1.03$. To efficiently solve the model with uncertainty, we rely on the FiPIt global algorithm proposed by Mendoza and Villalvazo (2020), which converges for parameters close to the parameter regions that deliver a unique equilibrium derived in Section 2.3.1. Hence, to analyze the role of a LtV limit between 0 and 1 we used a capital income share below the uniqueness threshold, specifically $\alpha = 0.02$. Although this level seems low compared to the standard 0.33 value, it is comparable to the values used in in studies that look at Sudden Stops in models with stock collateral constraint. For example Jeanne and Korinek (2019), Bianchi and Mendoza (2018), Korinek and Mendoza (2014).

The recursive representation of the problem of the household becomes:

$$\begin{aligned} V(a,b,s) &= \max_{\{c,b',a'\geq 0\}} u(c) + \beta E_{s'|s}[V(a',b',s')] \quad s.t. \\ c + R^{-1}b' + qa' &= (1-\alpha)y(s) + a(\alpha y(s) + q) + b \\ R^{-1}b' &\geq -\kappa q a'. \end{aligned} \tag{28}$$

3.1 Positive Analysis

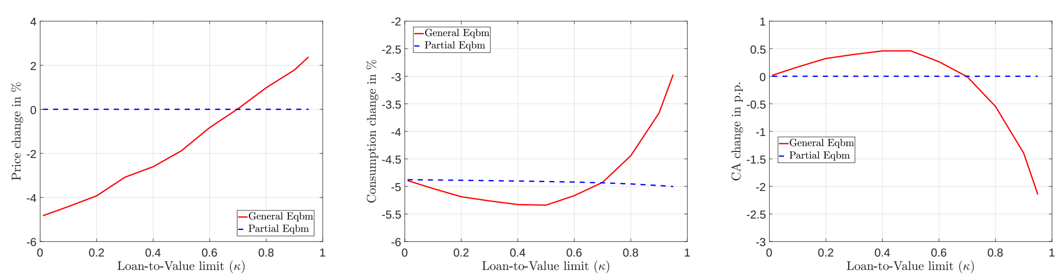
In this section, we analyze the quantitative implications of varying the loan-to-value (LtV) limit, κ, in an economy where the households are exposed to an aggregate shock to their endowment. The representative agent framework allows us to model the aggregate implications of endowment shocks, providing a clear lens through which to observe the dynamics of financial crises under different levels of LtV limits.

3.1.1 Impulse Responses

To explore these dynamics, we solve, simulate, and derive impulse response functions (IRFs) starting from the stochastic steady state for a range of LtV limits. Figure 5 illustrates the impact of a 2 standard deviation shock on key macroeconomic variables–namely asset prices, consumption, and the current account to GDP ratio–across different values of κ. The solid red lines in the figure depict the changes in these variables on impact. The impulse response were constructed in the following way. First we obtain the household's equilibrium decision rules and we simulate an economy without any shocks to the endowment to obtain the stochastic steady state for each level of κ. Since $\beta R < 1$, the stochastic steady state corresponds to a constrained bond position.[2] From that starting point then we simulate an economy that gets hit with a 2 standard deviations shock to their income process (Eq. (27)) and another economy with the same shock series starting on period 2 onward. Lastly, on Figure 5, we show the difference of both economies and report the response on impact for each level of κ.

Figure 5: Change on impact after a 2 s.d. shock

(a) Asset Price (b) Consumption (c) Current Account to GDP ratio



Our findings reveal a nonmonotonic relationship between the LtV limit and the economy's response to shocks, consistent with the theoretical predictions outlined in the earlier sections using the deterministic model. Specifically, as κ increases, we observe that the effects of the shock on asset prices, consumption, and the current account vary in a nonlinear fashion. This nonmonotonicity underscores the complex role that financial constraints play in amplifying or dampening

[2]This analysis also serves as validation for the assumption made in Section 2 about the marginally constrained bond initial conditions.

the impact of shocks in the presence of aggregate uncertainty and precautionary savings.

A crucial aspect of these results is the role of the economy's stochastic steady state, where $\beta R < 1$. This condition ensures that the economy operates in a constrained state, making the simulations directly comparable to the analytical results obtained earlier. The constrained state reflects the inherent vulnerability of the economy to shocks, particularly when borrowing limits are binding, and the LtV limit is a pivotal factor in determining the severity of the economy's response.

To further understand the mechanisms driving these results, we contrast the general equilibrium effects with partial equilibrium responses. The blue dashed lines in Figure 5 represent the outcomes when the asset price is held fixed at its stochastic steady state level, effectively isolating the role of pecuniary externalities. In this partial equilibrium scenario, the decline in consumption is relatively flat across different LtV limits, indicating that without the feedback loop generated by fluctuating asset prices, the economy's response to shocks is more muted.

This comparison highlights the critical importance of pecuniary externalities in shaping the overall dynamics. The nonmonotonic consumption behavior observed in the general equilibrium setting is largely driven by the interaction between asset prices and collateral constraints. Specifically, when the LtV limit increases, two opposing forces come into play. On one hand, higher LtV limits allow for greater indebtedness, making the economy more vulnerable to shocks and amplifying the drop in consumption. On the other hand, a looser collateral constraint mitigates the downward pressure on asset prices, dampening the overall impact of the shock.

In economies with low LtV limits, the vulnerability effect dominates, leading to a more pronounced decline in consumption. Conversely, in economies with high LtV limits, the mitigating effect on asset prices prevails, resulting in a less severe consumption decline. This interplay between vulnerability and mitigation is crucial for understanding the nonlinear responses of economies to financial shocks and underscores the nuanced role of LtV limits in crisis dynamics.

Overall, these quantitative results not only reinforce the theoretical predictions but also provide a richer understanding of the mechanisms at play. By illustrating how the LtV limit influences the economy's response to shocks through both direct and indirect channels, this analysis contributes to a more comprehensive view of the factors that drive financial instability and offers insights into the potential policy implications of varying collateral constraints.

3.1.2 Event Study

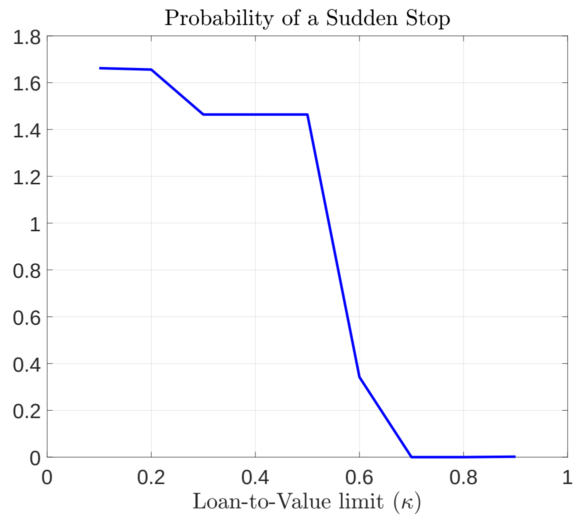
The impulse response analysis in the previous subsection examined responses to shocks starting from the stochastic steady state. While informative, this approach abstracts from two important features of actual crises: (i) crises occur from a distribution of initial states, not a single starting point, and (ii) the frequency of crises may vary with policy parameters. To address these limitations, we now conduct an event study analysis that endogenizes both crisis occurrence and initial conditions.

We simulate the economy for 110,000 periods for each level of κ and discard the first 10% to eliminate dependence on initial conditions. We identify a Sudden Stop episode as any period in which the change in the current account-to-GDP ratio exceeds 2 standard deviations above its historical mean–a definition commonly used in the empirical literature. This approach allows the initial bond position to evolve endogenously according to households' optimal saving and borrowing decisions, providing a more realistic characterization of crisis dynamics than impulse responses from an arbitrary starting point.

In terms of the frequency of crises, Figure 6 plots the probability of experiencing a Sudden Stop as a function of the LtV limit. The relationship is monotonically decreasing: economies with higher LtV limits experience fewer crises. This pattern reflects the role of precautionary savings in crisis prevention. When borrowing constraints are tight (low κ), households have limited ability to self-insure against adverse shocks, making crises more frequent. Conversely, when constraints are loose (high κ), households with higher borrowing capacity can manage debt more effectively, reducing crisis incidence.

This finding reveals an important tension in macroprudential policy design: tighter LtV limits increase crisis frequency but–as we show next–may reduce crisis severity. Policymakers face a

Figure 6: Probability of Sudden Stop Episodes by LtV Limit



Notes: Sudden Stops defined as current account reversals exceeding 2 standard deviations above the historical mean. Based on simulations of 99,000 periods for each κ value after discarding the initial 10% of the sample. Probability calculated as the frequency of Sudden Stop episodes in each simulation.

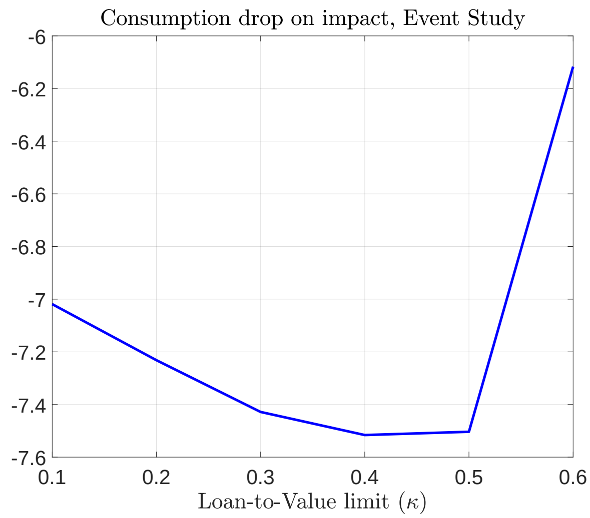
fundamental trade-off between preventing crises and limiting their damage when they occur.

Regarding crisis severity, Figure 7 examines the average consumption decline during Sudden Stop episodes for economies with different LtV limits. Consistent with our analytical results and impulse response analysis, the relationship exhibits a U-shape. For LtV limits in the range where crises occur with positive probability, consumption drops are most severe at intermediate κ values and smaller at both low and high extremes.

This pattern emerges even though initial conditions are endogenous and vary across simulations. The U-shaped relationship is not an artifact of starting from a particular state, but rather reflects the fundamental tension between the vulnerability effect (dominant at low κ) and the price stabilization effect (dominant at high κ) that operates regardless of the initial bond position.

We now examine how permanent adjustments to the LtV limit affect crisis severity within an economy. This analysis addresses a key question raised by the previous literature: while Bianchi and Mendoza (2018) characterize optimal *state-contingent* macroprudential policy that varies over the business cycle, less is known about how *permanent* changes in the regulatory stance affect crisis dynamics. Our contribution is to show that permanent adjustments to LtV limits implemented

Figure 7: Average Consumption Decline During Sudden Stop Episodes



Notes: Average percentage decline in consumption during Sudden Stop episodes relative to the pre-crisis level. Only includes κ values for which Sudden Stops occur with positive probability in the simulation.

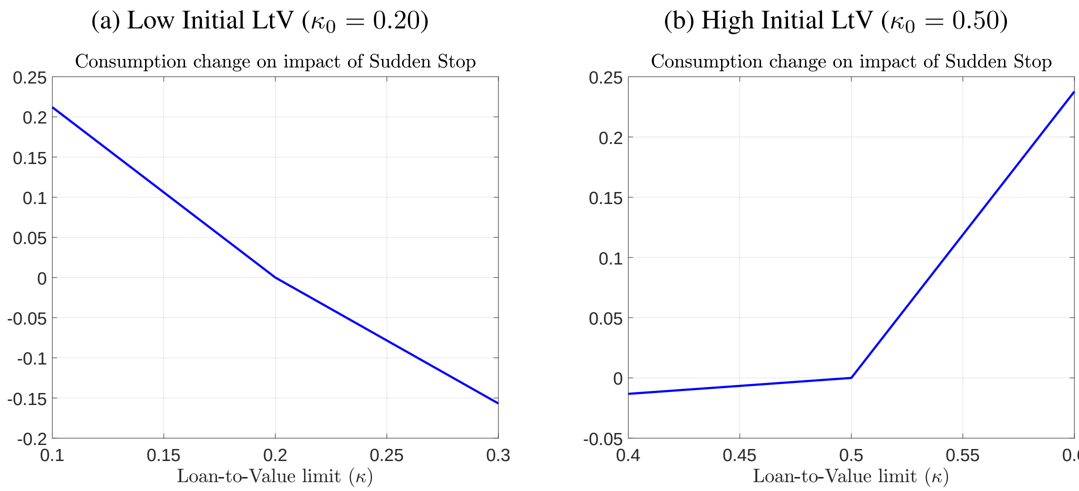
prior to crises have nonmonotonic effects on severity–the same policy change can be stabilizing or destabilizing depending on the initial level of κ. This finding complements the optimal policy literature by demonstrating that even the baseline level of regulation (not just its cyclical variation) has important consequences for financial stability.

Figure 8 presents this counterfactual experiment. For each crisis episode identified in the baseline simulation, we re-simulate the economy with an alternative LtV limit, holding fixed the sequence of shocks and the initial state at crisis onset. This controlled comparison isolates the causal effect of κ on crisis severity.

Panel (a) examines an economy with an initially low LtV limit ($\kappa = 0.20$). Permanently increasing the LtV limit amplifies crisis severity: the consumption decline becomes more pronounced as κ rises. This occurs because higher limits allow greater leverage accumulation, increasing vulnerability to shocks–the vulnerability effect dominates in this region.

Panel (b) presents the opposite case: an economy with an initially high LtV limit ($\kappa = 0.50$). Here, permanently increasing the LtV limit *dampens* crisis severity. The price stabilization effect now dominates: looser constraints reduce the amplification of asset price declines, cushioning the impact on consumption.

Figure 8: Effect of Permanent LtV Changes on Crisis Severity



Notes: Vertical axis shows the change in consumption decline (in percentage points) when the LtV limit is permanently changed from its initial value κ_0 to the value on the horizontal axis. Counterfactual simulations hold fixed the shock sequence and initial state at crisis onset. Negative values indicate that increasing the LtV limit amplifies the crisis; positive values indicate dampening.

These findings carry important implications for macroprudential policy: First, the effects of policy changes depend critically on the initial regulatory environment. The same policy action–loosening LtV limits by 10 percentage points–has opposite effects depending on where the economy starts. International policy coordination efforts that recommend uniform adjustments across countries may be misguided if countries differ in their initial κ levels. Second, there exists a "danger zone" of intermediate LtV limits where crisis severity is maximized. Economies in this region should consider either tightening substantially (to benefit from reduced leverage while recognizing this may increase the probability of a crisis) or loosening substantially (to benefit from dampened price effects), rather than maintaining intermediate values. Third, combining Figures 6 and 8 reveals the full policy trade-off: tightening LtV limits increases crisis frequency but may reduce severity (when starting from low κ), while loosening limits decreases frequency but may increase severity (when starting from low κ). Policymakers must weigh these offsetting effects based on society's preferences over crisis frequency versus severity. Fourth, our results complement the optimal macroprudential policy literature pioneered by Bianchi (2011) and Bianchi and Mendoza (2018). While that literature focuses on state-contingent policy that varies over the business cycle, we show that even the *permanent level* of LtV limits–which is often treated as a fixed structural feature–has important and nonlinear effects on crisis dynamics. Optimal policy likely involves both choosing an appropriate baseline κ (informed by our analysis of permanent effects) and implementing state-contingent adjustments around that baseline (as in Bianchi and Mendoza (2018)). The two approaches are complementary: our work informs the choice of the baseline regulatory stance, while the optimal policy literature informs how to adjust that stance cyclically.

In summary, the event study analysis validates that our main theoretical finding–the U-shaped relationship between LtV limits and crisis severity–is robust to: (i) endogenous initial conditions, (ii) endogenous crisis occurrence, and (iii) averaging over the full distribution of states rather than starting from a single point. Moreover, the counterfactual analysis demonstrates that permanent policy changes have causal effects on crisis severity that depend nonlinearly on the initial policy stance, with important implications for macroprudential regulation.

3.2 Normative Analysis

The positive analysis revealed nonmonotonic effects of LtV limits on crisis severity, but welfare analysis uncovers an important tension between long-run efficiency and short-run flexibility. We distinguish between *steady-state welfare*, which measures unconditional expected lifetime utility using the economy's ergodic distribution in the stochastic steady state for a given κ, and *transitional welfare*, which accounts for the utility gains or losses from changing κ, averaging across states using the initial kappa's ergodic distribution (similar to the methodology described in Bianchi and Mendoza (2018)). This distinction is crucial for understanding the political economy of macroprudential regulation and the challenges policymakers face in implementing optimal policies.

3.2.1 The Steady-State versus Transition Trade-off

Figure 9 shows that steady-state welfare is maximized at $\kappa = 0$–the no-debt capacity economy–and decreases monotonically as the LtV limit rises. This result reflects the well-known *overborrow-*

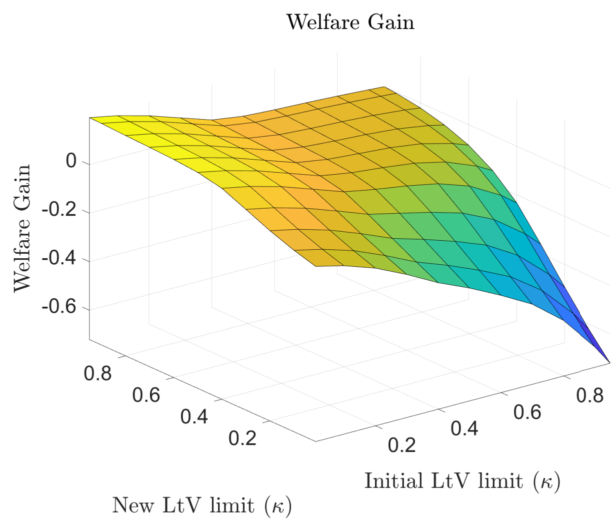
ing externality (Lorenzoni, 2008; Bianchi, 2011): when making borrowing decisions, households fail to internalize how their debt accumulation affects future asset prices and borrowing capacity. In a pecuniary externality framework, individual households take prices as given, but their collective borrowing decisions depress asset prices during crises, tightening constraints further and amplifying the downturn. Without access to debt ($\kappa = 0$), this externality is eliminated. Households self-insure through precautionary savings (accumulating positive bond holdings), achieving superior consumption smoothing and avoiding crisis states entirely. As the LtV limit increases, households accumulate more debt in good times, leaving them more exposed to adverse shocks and reducing long-run welfare.

Figure 9: Steady-State Welfare by LtV Limit

Notes: Welfare measured as unconditional expected lifetime utility in the stochastic steady state for each κ value. Vertical axis shows consumption-equivalent welfare relative to $\kappa = 0$. Based on ergodic distribution from simulations of 99,000 periods after discarding initial 10%.

Figure 10 reveals a striking contrast: transitional welfare from increasing κ is always positive, regardless of the initial LtV limit. This apparent contradiction with the steady-state results arises because permanent increases in κ generate short-term consumption booms that dominate long-run costs in present-value terms. The mechanism operates as follows. When the LtV limit is raised, households immediately exploit their expanded borrowing capacity, taking on additional debt to finance higher consumption during the transition. Although this additional debt reduces long-run consumption through increased debt service obligations, the immediate consumption gain is valued more highly due to discounting ($\beta < 1$). The present-value calculation weights near-term benefits heavily relative to distant costs, generating positive transitional welfare gains even though the new steady state has lower welfare than the old one.

Figure 10: Transitional Welfare Gains from Increasing LtV Limits



Notes: Welfare gain (in consumption-equivalent units) from permanently changing the LtV limit from an Initial LtV limit value axis to the New LtV limit value axis. Positive values indicate welfare gains during the transition despite lower steady-state welfare in the long-run. Calculated accounting for the full transition path.

The magnitude of transitional welfare gains varies with both the initial and final κ levels, but the sign remains positive throughout–increasing the LtV limit always generates short-run welfare improvements. This finding is consistent with Bianchi and Mendoza (2018), who show that fixed macroprudential taxes can produce welfare losses when computed conditional on initial states far from the constrained steady state, even though they improve welfare at the constrained steady state itself. Our analysis extends this insight to permanent changes in LtV limits rather than temporary tax interventions.

3.2.2 Policy Implications

The divergence between steady-state and transitional welfare creates a fundamental time-consistency problem for optimal macroprudential policy. Governments face strong political pressure to loosen

LtV limits because the benefits–immediate consumption increases enjoyed by current voters–are concentrated, visible, and experienced in the near term, while the costs–increased vulnerability to future crises–are diffuse, probabilistic, and realized only when adverse shocks materialize, potentially years later. This temporal mismatch explains why countries systematically allow financial deepening that reduces long-run welfare: incumbent governments can claim credit for expanding credit access and boosting consumption in the short run, while crisis costs materialize under future administrations. The transitional welfare gains documented in Figure 10 quantify these political rewards to financial liberalization. Although a social planner with commitment would choose $\kappa = 0$ to maximize long-run welfare, implementing such a policy faces formidable challenges: credibly preventing future relaxation despite large transitional gains, compensating current generations who must forego consumption to deleverage, and maintaining tight limits across political cycles and leadership changes.

Absent full commitment, second-best policies must balance multiple objectives while remaining politically sustainable. Our analysis suggests three key considerations. First, policymakers face an inherent tension between limiting overborrowing (favoring low κ) and allowing some credit expansion (favoring higher κ). Second, our positive analysis identified strong nonlinear amplification effects, with crisis severity maximized at intermediate κ values–roughly $\kappa \in [0.4, 0.6]$ in our baseline calibration. This "danger zone" exhibits the worst features of both tight and loose regimes: high enough leverage to amplify shocks, but not high enough to significantly dampen asset price declines. Economies in this range should consider either tightening substantially or loosening substantially, depending on which adjustment is more feasible. Third, as documented in Section 3.1.2, higher κ reduces crisis frequency but may amplify severity, creating a trade-off that depends on society's relative weights on preventing crises versus limiting their damage. While our analysis does not prescribe universal optimal values, it provides a rigorous framework for understanding the costs and benefits of different regulatory stances and highlights that intermediate LtV limits may be particularly problematic for financial stability.

3.2.3 Summary of Normative Findings

The welfare analysis reveals a fundamental challenge in macroprudential policy design. Long-run welfare is maximized by tight LtV limits that prevent overborrowing, but short-run political pressures push toward loose limits that generate immediate consumption gains. This time-consistency problem helps explain the persistent tendency toward excessive financial deepening observed empirically. Given these constraints, second-best policies should avoid the intermediate range where crisis amplification is most severe, choosing instead either tight limits (if commitment is feasible) or relatively loose limits (if political economy considerations dominate). The frequency-severity trade-off documented in our event study analysis further complicates optimal policy, requiring policymakers to balance multiple objectives with imperfect tools. While our analysis does not resolve these trade-offs definitively, it provides a rigorous framework for understanding the costs and benefits of different regulatory stances and highlights the importance of institutional design in sustaining welfare-improving macroprudential policies over time.

4 Model Validation

This section presents descriptive evidence validating the nonlinear predictions of our theoretical model. Using a panel dataset spanning advanced and emerging market economies, we estimate quadratic regressions with consumption-to-GDP ratios and current account changes as dependent variables during Sudden Stop episodes. Our findings confirm the U-shaped pattern for consumption declines and the mirror-image inverted-U pattern for current account reversals predicted by the competing vulnerability and price stabilization effects in our theoretical framework.

4.1 Data and Measurement

Our empirical analysis combines data from multiple sources to construct a panel of crisis episodes. We use the Sudden Stop episodes identified by Bianchi and Mendoza (2020), consumption-to-GDP ratios from the World Bank's World Development Indicators (The World Bank, 2024), and current account-to-GDP ratios from the External Wealth of Nations Database (Lane and Milesi-Ferretti, 2018). Our sample covers 65 crisis episodes across 41 countries during 1984-2015, with sample sizes varying across specifications due to data availability.

The key explanatory variable is the Financial Institutions subindex from the IMF's Financial Development Index ($FinDev$), which measures the depth, access, and efficiency of banks and financial intermediaries (Svirydzenka, 2016). This serves as our proxy for the LtV limit, κ, because it captures contract enforceability, creditor protections, and the ability to recover collateral value– precisely the institutional features that determine the fraction of collateral lenders can recover in case of default, making it a natural empirical counterpart to the theoretical κ.

We acknowledge that this mapping is imperfect. Financial development encompasses not only creditor rights and collateral enforcement but also financial depth, government efficiency, and regulatory quality. Alternative mechanisms could generate similar nonlinear patterns: very low financial development may produce a financial sector too small to generate significant amplification, while very high development could provide insurance and diversification channels absent from our model. Our analysis therefore documents a pattern *consistent* with the theory rather than definitively isolating the collateral constraint mechanism. Section 4.3 presents robustness checks using alternative proxies.

4.2 Empirical Specification and Results

We estimate a descriptive nonlinear regression designed to capture any quadratic relationship between our dependent variables and the Financial Institutions subindex measure during crisis episodes. We emphasize at the outset that our empirical analysis provides suggestive rather than conclusive evidence for our theoretical mechanism. The goal is to document whether cross-country patterns are *consistent with* our model's predictions, not to definitively establish causality or rule out alternative explanations.

The baseline specification takes the form:

$$Y_{i,t} = \beta_0 + \beta_1 \tilde{\kappa}_{i,t} + \beta_2 \tilde{\kappa}_{i,t}^2 + \text{Controls}_{i,t} + FE_t + \varepsilon_{i,t}, \tag{29}$$

where $Y_{i,t}$ represents either the year-on-year change in the consumption-to-GDP ratio (demeaned by country-specific averages to focus on crisis deviations) or the year-on-year change in the current account-to-GDP ratio for country i in crisis year t. The variable $\tilde{\kappa}_{i,t}$ is the Financial Institutions subindex measure, which we interpret as a proxy for the LtV limit. The control variables include an emerging economy dummy, real GDP growth (capturing downturn severity), and the deposits-to-GDP ratio (measuring precautionary buffers). We include time fixed effects (FE_t) to control for global factors affecting all countries in a given year, such as commodity price shocks, international financial conditions, or worldwide risk aversion.

The coefficients of interest are β_1 and β_2, which capture the linear and quadratic effects of financial development on crisis outcomes. Our theoretical model predicts $\beta_1 < 0$ and $\beta_2 > 0$ for the consumption equation, implying that consumption declines are most severe at intermediate levels of financial development (the U-shape). For the current account change equation, we expect $\beta_1 > 0$ and $\beta_2 < 0$, reflecting the mirror-image inverted-U pattern: current account improvements (capital flow reversals) are most pronounced at intermediate financial development levels.

Table 2 presents the estimation results. Column 1 reports results for the change in the consumption-to-GDP ratio as the dependent variable, while column 2 shows results for the change in the current account-to-GDP ratio.

Figure 11 plots the fitted marginal effects of financial development with 90% confidence bands coming from the quadratic regression estimates of Table 2. Panel (a) confirms the predicted U-shaped pattern for consumption declines, with the most severe drops occurring at intermediate financial development levels, and smaller declines at both extremes. Panel (b) shows the mirror-image inverted-U pattern for current account changes, with maximum adjustments at similar intermediate values, consistent with our theoretical predictions.

Table 2: Financial Development and Crisis Severity

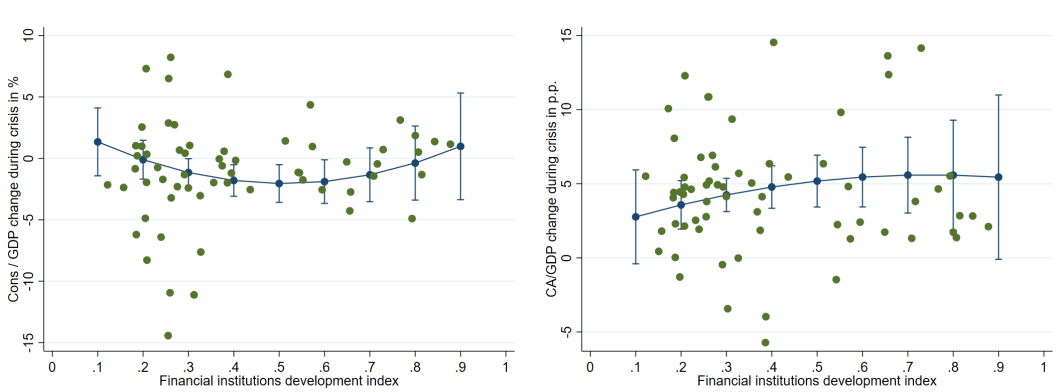
	Δ Consumption/GDP (1)	Δ Current Account/GDP (2)
$\tilde{\kappa}$	-20.50	10.07
	(12.25)	(15.50)
$\tilde{\kappa}^2$	20.05	-6.72
	(12.21)	(16.02)
Constant	-2.85	5.13
	(2.57)	(3.70)
Controls	Yes	Yes
Time FE	Yes	Yes
Observations	57	61
R^2	0.50	0.50

Robust standard errors in parentheses.
Controls include emerging economy dummy, real GDP growth and deposits-to-GDP ratio.
Sample: Sudden Stop episodes 1984-2015 from Bianchi & Mendoza (2020).

Figure 11: Nonlinear Effects of Financial Development on Crisis Outcomes

(a) Consumption-to-GDP Ratio

(b) Current Account-to-GDP Ratio



Notes: Fitted marginal effects computed from specification in column 1 (panel a) and column 2 (panel b) of Table 2. Bands represent 90% confidence intervals based on robust standard errors. Horizontal axis shows financial institutions development index. Panel (a) displays U-shaped pattern (most severe consumption declines at intermediate levels). Panel (b) displays inverted-U pattern (largest current account reversals at intermediate levels).

4.3 Robustness Checks

To verify robustness, we re-estimate equation (29) using two alternative proxies for κ: the World Bank's Rule of Law percentile rank (RoL), which captures contract enforcement and property rights, and the Regulatory Quality percentile rank ($RegQual$), which measures the ability of the government to formulate and implement regulations that permit and promote private sector development.

Table 3 presents the results. Both proxies yield negative linear coefficients and positive quadratic coefficients for the change in the consumption-to-GDP ratios, in line with the U-shaped relationship. The magnitudes vary across proxies–unsurprising given they capture different institutional dimensions–and data limitations (both indices start in 1996) result in fewer crisis episodes.

Table 3: Robustness: Alternative Proxies for Financial Development

	Consumption-to-GDP Ratio	
	(1) RoL	(2) $RegQual$
Proxy ($\tilde{\kappa}$)	-0.034	-0.073
	(0.30)	(0.23)
Proxy^2 ($\tilde{\kappa}^2$)	0.0003	0.001
	(0.003)	(0.001)
Controls	Yes	Yes
Time FE	Yes	Yes
Observations	29	29
R^2	0.49	0.52

Robust standard errors in parentheses.
Controls include emerging economy dummy, real GDP growth and deposits-to-GDP ratio.
Sample: Sudden Stop episodes 1984-2015 from Bianchi & Mendoza (2020).

The consistent quadratic pattern across diverse measures–financial sector depth, institutional quality, and governance–suggests the nonlinearity is a robust empirical feature. While no proxy perfectly captures the theoretical κ and each may reflect additional mechanisms, the convergence of results strengthens confidence in our theoretical predictions. The evidence suggests that the U-shaped relationship between LtV limits and crisis severity is observable in cross-country data, underscoring the importance of tailoring macroprudential policies to countries' institutional environments rather than applying uniform prescriptions.

4.4 Interpretation and Caveats

These empirical results should be interpreted as providing suggestive evidence consistent with our theoretical predictions, not as definitive proof of the collateral constraint mechanism. Several important caveats deserve emphasis.

Our financial development measures capture contract enforcement, creditor protections, and institutional quality–features that affect the collateral recovery fraction κ–but they also reflect financial depth, government efficiency, and regulatory frameworks that may operate through channels absent from our model. Moreover, the observed U-shaped pattern could arise from mechanisms beyond collateral constraints. For instance: (i) at very low financial development, the financial sector may be too small to generate significant amplification regardless of LtV limits; (ii) at very high development, insurance and diversification channels not modeled here could provide stabilization; (iii) nonlinear effects of financial regulation, supervision quality, or crisis management capacity could generate similar patterns. Furthermore, our analysis documents *correlations* between financial development and crisis severity, not causal effects of LtV limits. Countries differ along many dimensions simultaneously, and we cannot rule out omitted variable bias or reverse causality (e.g., countries experiencing severe crises might subsequently reform their financial systems). Finally, the relatively small number of crisis episodes limits statistical power, and our proxies vary more across countries than within countries over time, making it difficult to separate country-specific factors from financial development effects.

Despite these limitations, several considerations support the relevance of our findings. First, the U-shaped pattern is robust across multiple proxies (as shown in Section 4.3) that capture different institutional dimensions, suggesting it is not a measurement artifact. Second, the turning points are broadly consistent with our calibrated model's quantitative predictions. Third, the pattern is economically meaningful and aligns with case study evidence from major crises (e.g., the Asian Financial Crisis affected middle-income emerging markets more severely than either low-income economies with limited financial sectors or advanced economies with deep, diversified markets).

We view definitive causal identification as an important direction for future research. Ideal empirical strategies would include: (i) exploiting regulatory changes in LtV limits as natural experiments, using difference-in-differences or regression discontinuity designs; (ii) employing micro-level household or firm data that directly links actual LtV constraints to borrowing and consumption behavior; (iii) instrumenting for financial development using exogenous variation in legal origins, colonial history, or geographic factors; or (iv) conducting cross-country event studies with matched samples to control for confounding factors.

For the purposes of this paper, our cross-country evidence serves a more modest goal: documenting that the nonlinear patterns predicted by our theoretical model are empirically plausible and observable in aggregate data. This lends credibility to the mechanisms we identify and suggests they are quantitatively relevant for understanding real-world crisis dynamics, even if we cannot definitively rule out alternative explanations.

5 Conclusion

This paper investigates how loan-to-value borrowing limits affect crisis dynamics in equilibrium models with occasionally-binding collateral constraints. Our central finding is that the relationship between LtV limits and crisis severity is fundamentally nonmonotonic: tightening constraints reduces crisis amplification when limits are low but increases amplification when limits are high. This U-shaped pattern emerges from the tension between two opposing forces–higher LtV limits increase vulnerability through greater leverage (the vulnerability effect) but simultaneously reduce amplification by dampening asset price declines (the price stabilization effect). We establish this result through multiple complementary approaches: closed-form analytical solutions under perfect foresight, numerical analysis of the stochastic model comparing general and partial equilibrium, event study analysis that endogenizes crisis occurrence and examines both frequency and severity, welfare analysis revealing a time-consistency problem in macroprudential policy, and cross-country empirical validation documenting U-shaped patterns between financial development and consumption declines during Sudden Stops. The robustness of the U-shaped relationship across all these approaches–different assumptions about uncertainty, initial conditions, endogenous crises, and empirical proxies–establishes it as a fundamental feature of economies with collateral constraints.

Our findings have important implications for macroprudential regulation. First, the nonmonotonic relationship implies that uniform policy recommendations are misguided: the same adjustment–loosening or tightening LtV limits–has opposite effects depending on a country's initial regulatory stance. Second, there exists a "danger zone" of intermediate LtV limits (where debt is limited to approximately 50 percent of collateral value) where crisis amplification is maximized, suggesting policymakers should avoid these values. Third, our analysis reveals a frequency-severity trade-off: higher LtV limits reduce crisis probability but eventually amplify their severity, requiring policymakers to balance prevention against mitigation. Fourth, our welfare analysis uncovers a time-consistency problem: steady-state welfare is maximized when the LtV fraction limit equals zero (no debt capacity), but transitional welfare from loosening limits is always positive, creating political pressures toward excessive financial deepening despite long-run costs. This explains why countries systematically allow credit expansion that reduces welfare and suggests that sustainable macroprudential frameworks require institutional safeguards–independent regulatory agencies, transparent rules, or constitutional constraints–to maintain welfare-improving tight limits despite short-term political pressures.

Our work complements the optimal policy literature by showing that even the permanent baseline level of regulation (not just cyclical adjustments) has important nonlinear effects on stability. Our analysis makes clear contributions: by analytically characterizing the nonlinear relationship between LtV limits and crisis severity, demonstrating its robustness across methodologies, and documenting empirical patterns consistent with our predictions, we provide a rigorous foundation for macroprudential policy design. While our cross-country evidence is suggestive rather than conclusive–alternative mechanisms beyond collateral constraints could generate similar patterns–

the consistency between theoretical predictions and empirical regularities across multiple proxies strengthens confidence that the mechanisms we identify are quantitatively relevant for understanding real-world crisis dynamics. Future research employing more targeted identification strategies, such as natural experiments around LtV policy changes or micro-level data, would be valuable for definitively isolating the collateral constraint channel.

The central message is that the level of regulation matters nonlinearly–the same intervention can stabilize or destabilize depending on initial conditions, intermediate values may be particularly problematic, and political economy forces create systematic pressures toward excessive deepening. Effective frameworks must account for these nonlinearities and implement institutional safeguards against time-inconsistent policy relaxation.

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