Nonlinear Effects of Loan-to-Value Constraints[1]

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May 17, 2025

2025 Midwest Macro Meetings at Kansas City Fed

[1] The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

Motivation

- Financial crises often arise from the interplay between debt levels and credit constrains
- Since the GFC there has been an expansion of quantitative macro models that incorporate credit constraints. But not so much focus on their analytical properties
  - Few exceptions by Schmitt-Grohe and Uribe (2017, 2012)
- This paper delves into the intricate dynamics of financial crises, focusing on the role of the interaction between loan-to-value borrowing constraints and capital intensity in production

What we do

- Analyze macroeconomic vulnerability through a model-based approach
- We characterize the analytical solution of the eqbm asset price and consumption, which allows us to describe the parameter space regions where eqbm is unique, multiple or non-existent
  1. Unique equilibrium when the LtV limit is below the ratio of the discounted cost of capital relative to its return
  2. U-shaped relationship between consumption change and the LtV limit during crises.
  3. When uniqueness condition is not satisfied (i.e. high LtV limits and high capital intensity) financial markets may become unstable, leading to multiple or non-existent equilibria
- Extend the model with stochastic endowment and provide empirical validation of the model predictions

Model with Perfect Foresight

- Endowment SOE with representative HH that maximizes log consumption
- HH buys next period international bond holdings, b', that pay an exogenous interest rate R and also buys next period domestic asset holdings, a', with an endogenous price q
- Total endowment in the economy y, whose share of labor income is given by $(1-\alpha)$ and share to dividends income by α
- HH's credit is constrained by a loan-to-value collateral constraint with limit $\kappa \in [0,1]$

$$\begin{aligned} V(a,b) &= \max_{\{c,b',a'\}} \log(c) + \beta V(a',b') \quad s.t. \\ c + R^{-1}b' + qa' &= (1-\alpha)y + a(\alpha y + q) + b \quad \text{(Budget Constraint)} \\ R^{-1}b' &\geq -\kappa q a' \quad \text{(Collateral Constraint)} \end{aligned}$$

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- Economy marginally at the constraint $b = -R\kappa q$

$$
\begin{aligned}
b = b' = -\frac{R\kappa\alpha y}{R-1} \\
c = y(1-\kappa\alpha) \\
q = \frac{\alpha y}{R-1}
\end{aligned}
$$

Analytical Results and Characterization of the Equilibrium

- Financial crisis after an unexpected wealth-neutral negative shock

$$y_t = \begin{cases} y & \text{for } t \leq -1 \\ \gamma y & \text{for } t = 0 \text{ with } \gamma < 1 \\ \tilde{y} = y(1 + (R-1)(1-\gamma)) & \text{for } t \geq 1 \end{cases}$$

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- We can show that the equilibrium price is: $q_{-1} = \frac{\alpha y}{R-1}$, $q_1 = \frac{\alpha \tilde{y}}{R-1}$

$$Aq_0^2 + Bq_0 + C = 0$$

$$A = \frac{\kappa}{\beta}(\kappa + \beta - 1)$$

$$B = \kappa y \gamma \left(1 - \frac{\kappa\alpha}{\gamma(1-\beta)}\right) + \tilde{y}\left(1 - \kappa - \frac{\beta\alpha\kappa}{1-\beta}\right)$$

$$C = \frac{\beta\alpha\tilde{y}y\gamma}{1-\beta}\left(\frac{\kappa\alpha}{\gamma(1-\beta)} - 1\right)$$

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   - Crisis amplification or dampening from increases in the LtV from zero depend on the capital share for the price and unambiguous crisis amplification for drop in consumption:

$$\left.\frac{\mathrm{d}q_0}{\mathrm{d}\kappa}\right|_{\kappa=0} = \begin{cases} \geq 0 \text{ if } \alpha \leq (R-1)\frac{\gamma}{R} \\ < 0 \text{ if } \alpha > (R-1)\frac{\gamma}{R} \end{cases} , \quad \left.\frac{\mathrm{d}c_0/c_{-1}}{\mathrm{d}\kappa}\right|_{\kappa=0} < 0.$$

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3. When LtV limit is the maximum and unique ($\kappa = 1$ and $\alpha = (1 - R^{-1})\gamma$) then:
   - Crisis dampening in both the price and consumption as the maximum LtV is reached:

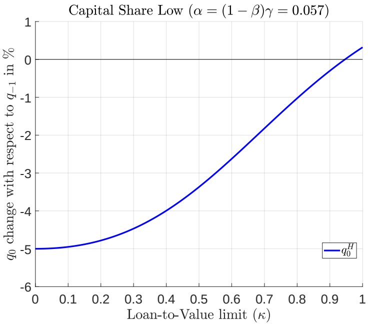
$$\left.\frac{\mathrm{d}q_0}{\mathrm{d}\kappa}\right|_{\kappa=1} > 0, \quad \left.\frac{\mathrm{d}c_0/c_{-1}}{\mathrm{d}\kappa}\right|_{\kappa=1} > 0.$$

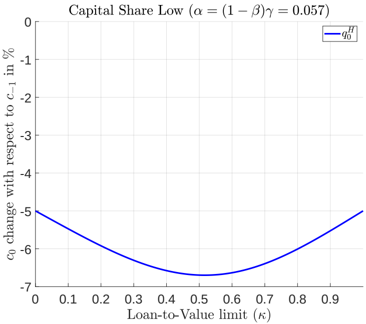
Numerical example

Parameter		Value
β, ($R = \beta^{-1}$)	discount factor	0.94
y	total endowment	1.0
γ	drop endowment	0.95
κ	LtV limit	$\in [0, 1]$
α	capital share	$(1 - \beta)\lambda = 0.057$

Numerical example: Unique Equilibrium

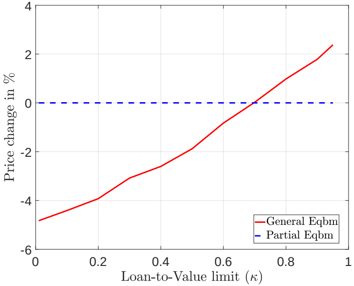
- The non-monotonic effect of loosening the LtV limit is largely driven by the interaction between asset prices and collateral constraints
  - Debt-deflation mechanism: fire-sales depress asset prices $\rightarrowtail$ tighter financial conditions
- Two opposing forces come into play with a higher LtV limit:
  - Greater indebtedness, economy more vulnerable to shocks: amplifying effect
  - Looser collateral constraint, mitigates downward pressure on asset prices: dampening effect

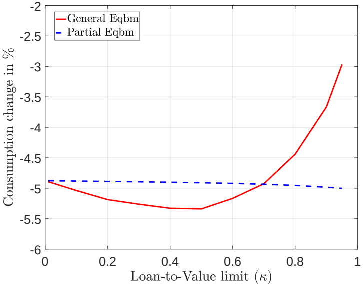




Model with Uncertainty (Partial and General Eqbm)

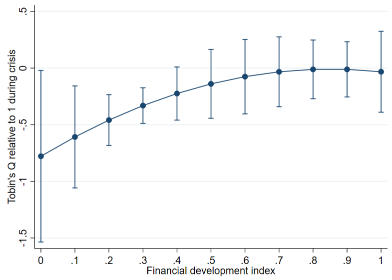
- Simple extension of the model where the income endowment, y, is stochastic and follows an AR(1) Markov process with $\sigma = 0.025$ and $\rho = 0.70$.
- Same parameters, lower $R = 1.03$ to guarantee a non-degenerate limit wealth distribution
- We solve, simulate, and do IRFs for different LtV limits (κ)
- Without the GE effect we get flat responses in individual consumption

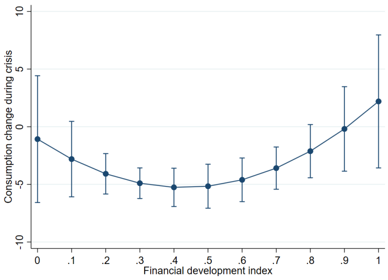




Model Validation

- Crises taken from Bianchi and Mendoza (2020) SS database, FD Index from the IMF and macro aggregates from the WB.
- Quadratic regression: $Dep_{i,t}\% = \beta_0 + \beta_1 FD_{i,t} + \beta_2 FD^2_{i,t} + \epsilon_{i,t}$
- These results provide empirical support for the theoretical framework. The effectiveness of financial development in mitigating crisis impacts is highly nonlinear





Conclusion

- This paper studies the intricate dynamics of financial crises, focusing on the role of the interaction between LtV borrowing constraints and capital intensity
- A unique equilibrium across all LtV limits exists when the capital share is low
  - U-shaped relationship between consumption change and the LtV limit during crises, balancing the benefits of increased borrowing capacity with the risks of heightened leverage
- When capital share increases, the model exhibits more complex dynamics, including the possibility of financial instability: multiple or non-existent equilibria
  - Reflecting the greater sensitivity of asset prices to shocks, especially in highly leveraged economies
- Model predictions are present in stochastic framework and validated with real-world data, bridging the gap between theory and practice

Appendix: Numerical example: Multiple Equilibria

- Multiple and non-existent equilibria